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A comparative Analysis of ICT Tools and the Mathematical Education of Blind and Visually Impaired people in Ireland, Poland, the Netherlands, and Neighbouring Countries

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EURO MATH
EDUCATION PLATFORM

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1 Introduction

1.1 Overview

One of the main goals of students' education is the acquisition of skills that will determine their functioning in the so-called community of knowledge and their success in the labour market. In 2006, the European Parliament (EP) described, defined and issued recommendations concerning the acquisition of key competences in individual subjects and general knowledge by young people completing their compulsory education. Among the four subject-defined competences are those pertaining to mathematics and basic scientific and technical skills, as well as IT competences. The need to acquire mathematical abilities concerns developing the aptitude to develop and use mathematical thinking in solving problems arising from everyday situations, with an emphasis on process, action, and knowledge. The development of IT competences is aimed at skilful and critical use of information society technologies (IST), and thus the use of basic ICT skills, including for acquiring mathematical competences. In particular, the use of ICT in mathematical education of blind and low vision students can greatly facilitate teacher's work and student's acquisition of mathematical knowledge.

The problem concerns not only blind people, but also a far greater number of people with low vision, amongst whom there is a very large variation in the level and type of visual disability, requiring the individualised application of supportive measures. Mathematical and physical expressions (hereafter known as mathematical formulas) and elements of mathematical graphics are non-linear, spatial objects that are difficult to read, recognize and edit by people with visual disabilities. It is necessary to convert formulas into a linear notation (such as AsciiMath [1]), and to provide access to diagrammatic information, it is necessary to use bespoke technology to produce tactile versions of graphs and other such artefacts. This is cumbersome due to the excessiveness of linear formulas in AsciiMath and in Braille notations, extending the time of mathematical operations, and it also requires additional special Braille equipment and software, as well as the adaptation of textbooks and other educational materials. Figure 1 presents examples of the inherent time extension required to write formulas in the Polish Braille Math Notation (BMN) in relation to the ASCII notation.

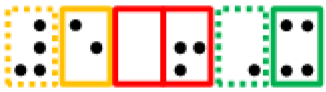
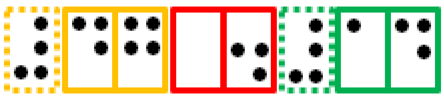
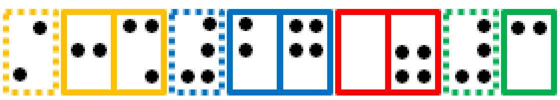
ASCII Formula	Length	Braille Formula	Length	Redundancy
$5 + x$	3 char		6 char	$\frac{6}{3} * 100\% = 200\%$
$67:14$	5 char		8 char	$\frac{8}{5} * 100\% = 160\%$
$\sqrt[3]{27} = 3$	5 char		10 char	$\frac{10}{5} * 100\% = 200\%$

Figure 1 Examples of the difference in the number of characters when writing formulas in BMN in relation to the ASCII notation

(Source: own work)

Though each of the countries represented in EuroMath has its own unique challenges when teaching mathematics, there are some general issues which arise irrespective of the location in which the content is being accessed. In this chapter, an overview is presented of the key issues underpinning a project such as EuroMath. It should be noted that this is not an exhaustive list, it is intended to give the reader a snapshot of the situation faced by those with sight-impairments who wish to read/solve mathematical equations, and explore diagrams such as graphs.

The overarching goal of the EuroMath project is to design an innovative multi-tool ICT platform to support teachers and learners with visual impairments in math instruction. Access to EuroMath solutions will be freeware. The project attempts to meet ICT needs that will level the playing field for learners with visual impairments in gaining math competencies aligned with the elementary and secondary education curricula in the partner countries. Both learners educated in the inclusive and mainstream settings, along with their teachers, will also benefit from the outcomes of the project.

Previous work, entitled PlatMat [2] is the foundation for the projected EuroMath model. PlatMat was developed as a result of two research projects conducted between 2014 and 2017. The PlatMat supports math instruction and is only available at present for Polish learners and teachers. It received two international innovation awards (Tell Us Awards, ITEX 2015).

Teachers and learners who are blind or have low vision attending elementary and secondary education in Poland, the Netherlands and Ireland will participate in the project. Representatives of institutions and organizations that serve people with visual impairments in mainstream and special education, research and innovation institutes will also be involved on an ad-hoc basis. Project Activities will be conducted in close partnership with teachers and their learners with visual impairments in Partner countries to ensure the results meet the particular needs and expectations of the target groups. The quality and usability of the EuroMath tools will be verified by the end users during assessment and validation activities.

The project expects to increase the awareness of the needs of blind and visually impaired learners with respect to mathematics, and also hopes to make a contribution to best-practices in the education of this demographic.

1.2 Key Issues

One of the key decisions which must be made when considering the design of a suite of tools to provide access for blind and visually impaired people is the means of presenting the information to the target user. In order to implement this ideal, the notion of what information to present needs to be decided on first, followed by how to present this material. It is therefore important to understand the reading process, in order to fulfil the dual purpose of determining both what and how to present the relevant information to the user.

A key feature which is present in the visual reading process is the role of the printed page. This medium affords the reader not only the facility to act as an external memory, but also facilitates a highly refined control over the flow of information. In his Ph.D. thesis [3], Stevens states that Raynor

describes reading as: "…the ability to extract visual information from the page and comprehend the meaning of the text" [4, p. 23]. Stevens also tells us that reading can be divided into three main domains:

1. The process of understanding what has been read.
2. The input of information from a physical, external source, into the reader's memory via the visual system;
3. The recognition of words and their integration into higher level structures such as sentences.

Unlike linguistically-oriented material, the presentation of mathematics in a manner which is accessible poses significant challenges. Karshmer et al. [5] highlight the two-dimensional nature of visual or printed mathematic equations, which is difficult to convey through both linear systems of synthesized speech and Braille. The spatial representation of equations can encode essential semantic information very necessary to understanding the mathematic construct.

The primary issue when translating mathematics from print to Braille or audio-based output is that of transforming a two-dimensional representation, where the spatial arrangement of symbols juxtaposed with white space gives rich semantic and syntactic understanding, into a linear presentation. Printed notation provides a persistent visual cue which is effectively non-temporal, however, audible representations are necessarily fleeting due to the inherently temporal nature of sound. Sighted users can therefore utilize printed material as a form of external memory and do not need to memorize the structure and layout of an equation. This conclusion is supported by the results of a series of cognitive experiments examining equation reading in sighted users, conducted by Gillan *et al* [6], which found that sighted subjects process operators and numbers more intensively than parentheses. This is perhaps unsurprising as the spatial structure of the equation (which is implied through the use of parentheses and other graphical symbols and delimiters) is unambiguous and persistent when presented visually. This suggests that working with mathematical material in a non-visual medium will result in an inevitable increase in cognitive load as this structural information must now be held in memory. This strongly implies that any method of presenting the spatial structure of an equation in a non-visual manner must be as easy as possible to cognitively process.

This brings its own unique challenges. As well as transforming the material into a spoken or Braille representation, the developer of any system must supply mechanisms to enable the user to read mathematics in its entirety, or indeed to decompose the material into chunks for easy navigation. That so many attempts have been made to solve this particular problem is a testament to its intransigence.

As well as the technical challenges, there are also socio-cultural issues which must be taken into consideration. In many countries, opportunities for blind and visually-impaired people to participate in the higher levels of education, particularly in STEM (Science, Technology, engineering and Mathematics) can be very limited. There are many reasons for this. Historically, many sighted teachers have held on to the belief that those with a visual disability could not engage with this type of material. As a consequence, blind and visually impaired children have been actively discouraged from studying subjects such as mathematics. Secondly, the teaching infrastructure does not readily lend itself to the extra time needed to assist blind and visually impaired learners in apprehending STEM content. This can be due to large numbers in classes, a lack of knowledge on the part of the teachers, a lack of support from professionals who are versed in matters pertaining to how those with visual disabilities can engage with this material, or a whole host of other reasons too numerous to mention. It is against the background of these technical and societal challenges that the EuroMath project has been conceived, and it is the hope of all involved that this work will improve the educational and employment prospects of a group of people who, by virtue of the lack of opportunity in this domain, are extremely under-represented.

1.3 Report Structure

This report contains a description of the manner in which blind and visually impaired individuals interact with mathematical content in the three partner countries conducting the EuroMath project, namely: Ireland, Poland and the Netherlands. It also contains information pertaining to neighbouring countries such as Belgium, the Czech Republic, Germany, Slovakia, and the United Kingdom. Each section contains details, where appropriate and relevant, on the following topics:

1. The educational system in each of the partner nations;
2. The mathematics curriculum in the partner nations;
3. The examination/assessment structures in the partner nations;
4. How mathematics is taught to blind and visually impaired people in the partner nations, if and where such information is available;
5. The manner in which blind and visually impaired people interact with mathematics;
6. The manner in which this demographic interacts with diagrams such as graphs etc.

The reader is asked to bear in mind that, owing to the idiosyncrasies of the topic, and

how each country deals with it, the structure of each of the sections is not identical. The authors have striven to provide analogous information; however, it has not always been possible to do so. This report concludes with a brief chapter outlining the findings which may be drawn, and also with a commentary on how the situation in the various countries will affect the future work of the EuroMath project.

In this document, various terms are used to denote various differences in sight. For purposes of clarity, the term "blind" is used to indicate users with no vision, who rely on assistive technology such as screen readers to interact with ICT products. These users primarily rely on Braille or spoken output to obtain their information. The terms "vision-impaired", "visually-impaired" , "partially sighted", and "low vision" are used interchangeably. These terms describe users who have residual vision, and primarily use magnification software to interact with ICT products. These users may also use speech output, and occasionally Braille. No single term has been chosen, as each are used in the partner countries to describe this demographic.

2 Teaching Mathematics to Blind and Vision-Impaired students: The Irish Context

Before embarking on a discussion of the manner in which blind/vision-impaired students in Ireland interact with mathematics, it is first necessary to highlight that quantitative data pertaining to this demographic is not available. Thus, the information provided in these pages has been gleaned from feedback supplied by those involved in teaching this subject to the students in question. It is not intended here to imply any rigorous scientific analysis, as the sample size of those providing the information is too small. However, the qualitative data given here is no less valuable, as it has been obtained from persons who have spent a lifetime in the teaching of mathematics to blind/vision-impaired students. In order to obtain the information, a questionnaire was disseminated to “visiting teachers” who have the responsibility for working with the classroom teacher to ensure equal access to educational opportunities for blind/vision-impaired students. At the time of writing, only one of the 11 visiting teachers has supplied information. However, as this individual is the most senior member of this cohort, and has worked closely with relevant authorities to define the rules and frameworks by which mathematics is taught to this demographic, the input obtained is deemed to be highly accurate. Another source of input for this report was a focus group held on 13th April 2018 with teachers of mathematics from Pobailsoil Rosmini, which is the only specialist school to offer secondary education to blind/vision-impaired students in Ireland. Five teachers were present and the information was gathered through informal conversation. Finally, input has been received from the Reading Services department of Childvision [7] which has the responsibility for producing all school textbooks for blind/vision-impaired children in Ireland. This author is indebted to everyone involved in the teaching of mathematics in Ireland who have given their valuable time and energies to ensure that this report may be as accurate as possible. It is recognised that the numerical, and other, data found in other parts of this report is lacking in the Irish context, however the Department of Education and Skills have advised that given the numbers of students involved in this somewhat niche demographic are so small, that to provide the types of numerical data requested would, without any difficulty, identify the students in question and thus would be a violation of their data-protection rights under the new GDPR legislation. Finally, the primary focus of this section of the report is on the Irish context. Information is provided on the situation in the United Kingdom. Neither time nor project resources enabled the acquisition of a significant quantity of data on the neighbouring country. Rather, summary information is provided by way of comparison with Ireland, and the reader is directed to the relevant bibliographic entries where more detail may be found.

According to Section 2 of the Education for Persons with Special Educational Needs (EPSEN) Act 2004 requires that:

“A child with special educational needs shall be educated in an inclusive environment with children who do not have such needs unless the nature or degree of those needs of the child is such that to do so would be inconsistent with:

- The best interests of the child as determined in accordance with any assessment carried out under this Act
- The effective provision of education for children with whom the child is to be educated."

In general, educational provision for children with special needs is made:

- In special schools;
- In special classes attached to ordinary schools;
- In integrated settings in mainstream classes.

As is the norm in other countries, teachers working in special schools are responsible for the entire programme of education undertaken by the students. In mainstream settings, the classroom teacher is supported by a specialist expert who offers guidance on all aspects of the child's needs as they pertain to the visual disability. The individuals who take responsibility for these activities are known as "Visiting Teachers" and only periodically come into contact with the students. The level of support varies according to the child, and the region of Ireland in which the education is taking place. In the UK, the same function is fulfilled bya "Qualified Teacher of the Visually Impaired" (QTVI). One of the central components of the Education for Persons' with Special Educational Needs (EPSEN) Act (2004) was the introduction of Individual Educational Plans (IEP's) which would allow teachers access to information regarding the individual needs of the student. This could include various aspects such as learning aids, teaching strategies and appropriate modification to learning material. However, as this aspect of the Act has not yet been enacted, the implementation of IEP's remains optional and are not obligatory in Ireland.

2.1 The Education System

In Ireland, the education system consists of three distinct phases. They are:

- Primary;
- Second-level (also known as secondary);
- Third-level (consisting of college/university, or further/vocational training).

It is required that children attend school between the ages of six and sixteen, or until a minimum of three years of second-level education have been completed. Pre-school education is usually provided by privately funded childcare facilities or providers. Children do not have to attend school until the statutory age, however it is the norm for children to start school the September following their fourth birthday. Four-year-olds and five-year-olds are enrolled in the junior or senior infant classes. Once these have been completed, they then proceed to study for six more years in primary school. These classes are numbered from 1-6. The curriculum for primary education covers the following key areas: Language mathematics; social, environment and scientific education; arts education including visual arts music and drama; physical integration; social personal and health education.

The mathematics curriculum takes place from Junior infants to 6th class. It identifies the skills to be developed, and is taught in a manner which encourages creativity and problem-solving. It is taught through the medium of discussion, and the use of materials and hands-on experience. The main aim is to develop skills in mathematical language and thinking, and to encourage mathematical discourse. The key objectives of this portion of education are:

- Application of concepts, and ensuring problem-solving capabilities;
- The ability to both communicate and express mathematical concepts and constructs;
- Integrating the learning of concepts in one strand, and connecting these ideas to those from another area of the curriculum;
- Development of capacity in both reasoning and implementation.
- Applying the concepts learned, and developing the ability to recall them once the learning has been completed.

The content for children's learning is broken into various key areas:

- Number: This begins with four strands: Classifying, matching, comparing and ordering. At infant level this includes counting and analysis of numbers. During first and second classes, place-value, operations and fractions are taught. Decimals appear in third class, and percentages in 5th.
- Algebra: This topic is formally recognised at all levels and covers patterns, sequences, number sentences, directed numbers, rules and properties, variables and equations.
- Shape and Space: as a strand, shape and space explores spatial awareness and its application. Its focus is on real-life situations, and includes units dealing with two-dimensional and three-dimensional shapes, symmetry, lines and angles.
- Measures: this strand consists of six units which aim to provide instruction in all aspects of measuring and estimation. The topics are made up of material pertaining to length, area, weight, capacity, time and money.
- Data and Chance: In this part of the curriculum, material is presented which includes interpreting and understanding visual representations. Chance promotes thinking, discussion and decision-making and is introduced to children in the form of games and sporting activities.

Second-level education consists of a three-year junior cycle followed by a two-year or three-year senior cycle depending on whether an optional Transition Year is taken following the Junior Certificate examination. Students generally commence the junior cycle at the age of 12. The Junior Certificate is taken after three years. Junior Certificate Mathematics aims to "…develop the mathematical knowledge, skills and understanding needed for continuing education, for life and for work". [8] Its key objective is to develop the skills of dealing with mathematical concepts in context and applications, as well as in solving problems. It also aims to support the development of literacy and numeracy skills, and foster a positive attitude to mathematics in the learner. There are several levels of mathematics which the learner may study. These are known as Ordinary level, and Higher level (sometimes called “honours”). Very few blind students attempt honours, or higher level mathematics. The reasons for this are complex. Firstly, the time taken to study for the Higher Level mathematics exam is considerable. Students and teachers feel that to devote such an amount of extra effort to this subject would detract from the grades in other subjects. Secondly, Irish learners only receive ten minutes per-hour extra in state examinations irrespective of the subject. Given the extra time needed to both solve and read mathematics, it is considered by many that the amount of available time is wholly insufficient to enable the blind or vision-impaired student to complete the examination to the standard required.

The curriculum consists of the following strands:

- Statistics and probability;
- Geometry and Trigonometry;
- Number;
- Algebra;
- Functions.

As the content of both the Ordinary and Higher level variants of this syllabus are extensive, the reader is referred to [9, 10] where all relevant details are available.

During their final two years in the senior cycle, students take one of three programmes, each leading to a State examination - the established Leaving Certificate, the Leaving Certificate Vocational Programme or the Leaving Certificate Applied. The established Leaving Certificate is the main basis upon which places in universities, institutes of technology and colleges of education are allocated. The Leaving Certificate Vocational Programme differs from the established Leaving Certificate in placing a concentration on technical subjects and including additional modules which have a vocational focus.

The Leaving Certificate Applied Programme has as its primary objective the preparation of participants for adult and working life through relevant learning experiences. These aim to develop the following areas of human endeavour: spiritual, intellectual, social, emotional, aesthetic and physical. The Leaving Certificate Applied is not recognised for direct entry to third-level courses but it can enable students to take Post-Leaving Certificate courses. These courses are typically vocational in nature, and act as a bridge from secondary education to third level. It is worth commenting at this juncture that, as the numbers of blind and vision-impaired students in Ireland are so small, access to data detailing the numbers of students taking each of the variants of the Leaving Certificate is unavailable. Whilst anecdotal evidence could be obtained from other sources, to do so would place this author in contravention of the newly established GDPR regulations. Therefore, very little by way of numerical data is included in this portion of the report, as to do so would be mere speculation.

The objectives of Leaving Certificate Mathematics are based around key principles. It is designed to foster conceptual understanding—comprehension of mathematical concepts, operations, and relations. Secondly, it is designed to provide skills in carrying out procedures flexibly, accurately, efficiently, and appropriately. Thirdly, it aims to provide learners with the ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts. By the time learners have completed the Leaving Certificate, they should have the capacity for logical thought, reflection, explanation, justification and communication.

The Leaving Certificate curriculum comprises five strands:

- Statistics and Probability;
- Geometry and trigonometry;
- Number;
- Algebra;
- Functions.

It should be noted here that an increased emphasis has been placed on Euclidean geometry, and applied probability and statistics, which are topics that are dominated by visual content. In contrast, there has been a de-emphasis on calculus and linear algebra with vectors and matrices being completely removed from the syllabus; all of which are less visual in nature. Therefore, it could be argued that rather than making the Mathematics curriculum more inclusive these changes may have made it more exclusionary for blind/vision impaired students. In fact, this was an argument put forward by AHEAD [11] who asserted that with the increase of visual dimensions within the new syllabi for Project Maths that blind/vision impaired students are now facing further challenges accessing and engaging with the curriculum.

As the content of both the Ordinary and Higher level variants of this syllabus are extensive, the reader is referred to [8] where complete details are provided.

Across the UK there are five stages of education: early years, primary, secondary, Further Education (FE) and Higher Education (HE). Education is compulsory for all children between the ages of 5 (4 in Northern Ireland) and 16. FE is not compulsory and covers non-advanced education which can be taken at further (including tertiary) education colleges and HE institutions (HEIs). The fifth stage, HE, is study beyond GCE A levels and their equivalent which, for most full-time students, takes place in universities and other HEIs and colleges. It is important when considering the case of the UK, that although it is considered one country, there are regional differences in the various educational systems which operate within this jurisdiction. It is beyond this necessarily brief treatment to delve deeply into the intricacies of each of the variants. Thus, a superficial glance at this system is provided here for reference and comparison purposes only. It should also be borne in mind that, owing to centuries of shared history, there are many similarities between the education systems in both countries. The terminology used to describe them is somewhat different, however in all practicality they are analogous.

The primary stage covers three age ranges: nursery (under 5), infant (5 to 7 or 8) and junior (up to 11 or 12). In Scotland and Northern Ireland there is generally no distinction between infant and junior schools. In Wales, although the types of school are the same, the Foundation Phase has brought together what was previously known as the Early Years- from 3 to 5-year-olds – and that covering the education of children from 5 to 7-year-olds to create one phase of education for children aged between three and seven. In England, primary schools generally cater for 4-11 year olds. It is usual to transfer straight to secondary school at age 11 (in England, Wales and Northern Ireland) or 12 (in Scotland), but in England some children make the transition via middle schools catering for various age ranges between 8 and 14. Depending on their individual age ranges middle schools are classified as either primary or secondary.

The major goals of primary education are achieving basic literacy and numeracy amongst all pupils, as well as establishing foundations in science, mathematics and other subjects.

There are a variety of schools available at the secondary level in the UK. In England, Comprehensive schools largely admit pupils irrespective of educational aptitude or ability. They cater for all the children in a neighbourhood, but in some areas they operate alongside others for example grammar schools which only accept pupils who have attained a certain standard at primary level.

At the end of this stage of education, pupils are normally entered for a range of external examinations. Most frequently, these are GCSE (General Certificate of Secondary Education) in England, Wales and Northern Ireland and Standard Grades in Scotland, although a range of other qualifications are available. In Scotland pupils study for the National Qualifications (NQ) Standard grade (a two-year course leading to examinations at the end of the fourth year of secondary schooling) and NQ Higher grade, which requires at least a further year of secondary schooling.

"The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that

  pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately;

- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language;
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions." [12]

The mathematics curriculum covered in the UK is very similar to that taught in Ireland, and the reader is referred to [12, 10] for a detailed explanation of the content.

2.2 Assessment and Examination

It has been recognised that examinations in standard formats can pose challenges for blind/vision impaired students [13] In an effort to remove these challenges blind/vision impaired students are entitled to reasonable accommodations in state examinations. In Ireland, these accommodations are applied for through the scheme of Reasonable Accommodations at the Certificate Examinations (RACE). RACE assists candidates with Special Educational Needs (SEN) to demonstrate their knowledge and skills in state examinations while maintaining the integrity of the assessment and applying the same standards of achievement as applies to all other candidates [14]. Accommodations available to blind/vision impaired candidates include:

- Extra time (ten minutes for every hour, but not exceeding thirty minutes);
- Access to enlarged exam paper A3 size;
- Printed versions of modified exam papers;
- Braille versions of modified exam papers;
- Reader or reading assistant, writing accommodation in the form of a Word Processor, recording device, scribe [14].

It is worth taking a moment to explore the types of modification which may be made to examination papers in Ireland. On the modified versions of the examination papers diagrams and other images may be removed or simplified and tasks which involve drawing diagrams are substituted with other activities of a similar demand. For the papers in Braille format, tactile diagrams with Braille labelling are supplied. Candidates can have access to modified papers in both print (A4 or A3 size) and Braille format, however, cannot have access to both modified and unmodified exam papers. Candidates who avail of the modified papers will have an explanatory note accompanying their grade stating that "all parts of the examination in this subject were assessed except for the testing of graphical skills in the written papers." [14] It can be understood from these statements from the SEC that modifications that are being made to the Mathematics papers are exclusively related to the representation of diagrammatical information.

In the UK, the manner in which examinations are handled is very similar. The papers are modified by so-called "exam modifiers". When modifying the papers, the following criteria should be adhered to:

"

- The amended question must assess the same skills, knowledge and concepts as the original question in the print paper and enable the candidate to meet the same assessment objectives in National Curriculum subjects.

- The question should be of an equivalent level of difficulty as the original.

- Any alteration should preserve the balance of the original examination paper in terms of both the content and the weighting of questions.

- A modified question should not require candidates to spend a disproportionately large amount of time to gain relatively few marks.

- Where modification of an existing question is not possible, the modifier may propose a replacement question which attempts to meet the same assessment criteria for approval by the Awarding Body. In the event that a replacement question is deemed unacceptable by the Awarding Body, the modifier and Awarding Body should consult to determine what action will be taken, with due consideration to ensure the minimum of disruption or confusion for the candidate."

Perhaps the most important aspect of providing special assistance for blind/vision-impaired students in an examination setting is ensuring that they have an adequate amount of time to answer the questions to a satisfactory standard. Accordingly, there are mechanisms to ensure that this is provided. In the UK, blind students are permitted extra time in increments of 25%, 50%, or greater if the need arises. This is in stark contrast to the situation in Ireland (described above) where only ten minutes extra per hour are allocated; to a maximum of 30 minutes. In subjects such as mathematics (or other STEM-related disciplines) this would seem to be wholly inadequate.

For more detail on the manner in which examinations are conducted in the UK, please refer to [15].

2.3 Teaching Methodologies

Teaching and learning methodologies have been linked to blind/vision impaired students' ability to access the Mathematics curriculum [11, 16, 17, 18] The consensus of these authors seems to be that flexible and supportive teaching strategies are essential to ensure that blind/vision impaired students can reach their full potential when engaging with the Mathematics curriculum. It is vital that teachers understand the individual needs of the blind/vision impaired students they are teaching as these needs vary depending on the visual disability i.e. blindness or low vision [11]

When teaching the Mathematics curriculum, teachers need to make modifications to the curriculum and their teaching methodologies in order to meet the unique needs of blind/vision impaired students in their classroom [19] In [17] The authors have identify that the teaching methodology frequently implemented in the Mathematics classroom, "chalk and talk" which primarily focuses on what the teacher is saying and worked examples on the board impacts negatively on blind/vision impaired students' engagement with the curriculum and academic achievement. Taking in a large amount of spoken Mathematics information, without reference to what is happening on the board places a high demand on memory [20]. It was reported in [17]that as most mainstream teachers' lack any knowledge of Braille and Braille Mathematics notation this created an additional barrier for visually impaired students' specifically when engaging with the Mathematics curriculum.

Participants in the study found in [17]expressed the opinion that it was extremely beneficial when they had access to material in accessible formats prior to each lesson but unfortunately this was not always the case. This left students feeling like they were always playing catch up. Students who solely relied on access to Braille in order to read or solve mathematical equations where often left without key materials for extended periods of time.

When teaching mathematics, teachers employ a range of technologies. Laptops and tablets are utilised in conjunction with an electronic whiteboard. Interestingly, it emerged during the data-gathering phase of this research that there is a considerable difference between the methodologies used in mainstream schools compared to those used in the Special Educational setting. In the mainstream environment, the teacher will employ the laptop and/or tablet device to display various types of material on an interactive whiteboard. For example, the teacher may display samples of the relevant textbook, and demonstrate the worked examples found therein. The teacher will also highlight solutions to problems involving graphs using graphical calculators, or display videos from services such as YouTube. It is noteworthy that software such as Desmos [21] does not seem to be used by those who responded.

In the school focussing on Special Education, a far less technical approach is taken. Here, the teacher employs the more traditional approach of writing on a black/white board, and talking through the problem. It should be noted however that the reason for this may not be due to a difference in teaching approach; rather the specific issues faced by the school itself. Several years ago, the school buildings were severely damaged by a weather event and resulting fire. As a consequence, they are now located in temporary accommodation where many of the facilities available to other mainstream schools are simply unavailable. It will be interesting to revisit this topic in the coming years when the new premises have been built, and the necessary equipment and facilities are available.

In terms of infrastructure, the level of availability is extremely variable. Owing to the lack of rural broadband in Ireland, the presence of networks in schools cannot be guaranteed. Thus, the manner in which technology is used across the education sector can be dependent on the location of the school. Also, the level of IT literacy amongst teachers is also variable. Some teachers embrace technology, while many others favour the old-fashioned "chalk-and-talk" form of information presentation.

2.4 Access to Mathematics for Blind students

In order to understand the methods used by blind students to access mathematics in Ireland, it is first necessary to digress and outline how materials are provided. As it states in [7] ”National Braille Production at Childvision was established to meet the educational needs of children with a visual impairment attending either mainstream or special education at both primary and secondary level. Our Braille and alternative format transcription service was officially opened on the 1st of September 2000.”.

It is a national service which provides access to educational material by transcribing into a range of formats accessible to children with a visual impairment.

The formats currently catered for are: Braille, tactile diagrams, MOON, Large Print, DAISY books (accessible mp3 files) and customised text files for use with either speech output, magnification software or on different electronic braille output devices. All transcriptions follow recognised national and international guidelines.” [7]

It is worth noting that as well as providing materials in Braille, this organisation now operates a service (known as “Online Bookshelf”) which enables those who are registered to download materials onto devices which support digital formats such as DAISY [22]. In order to acquire materials from this service, a request must be made through the Visiting Teacher who is charged with working with the child in question.

The majority of mathematical content is produced using the Unified English Braille (UEB) code [23]. Whilst electronic Braille devices are becoming more prevalent, all children are provided with paper-based copies of all school textbooks. The process of producing materials in accessible formats for blind/vision-impaired students is made more complex, as there is no requirement on the part of the publishers to provide the books in digital (source) form. Consequently, the Reading Services division of Childvision [7] must:

1. Purchase paper copies of the materials;
2. Scan those into Microsoft Word format;
3. Make any necessary corrections manually.

For textual content this process is arduous enough, however in the case of mathematical expressions it is made even more cumbersome by the fact that the Optical Character Recognition (OCR) software in use does not handle this type of content. Therefore, each formula must be manually entered using the Equation Editor found in Microsoft Word. Once this process has been completed, the textbook may then be translated using the commercially-available Braille translation software.

The reason these details are important is that students can often be left waiting for their textbooks to be produced. Unlike other countries, there are a plethora of different books, from a variety of publishers, which cover the content prescribed in the mathematics curriculum. The lack of information available to the blind students forms a significant barrier to good performance in this subject.

In the main, the devices utilised by blind students to access mathematics both in Ireland and the UK are similar to those used in other countries. It will come as no surprise that this is the case, as the range of Assistive Technology available is comparable throughout the world. In addition, many of the companies who develop Assistive Technology for blind and vision-impaired people designate Ireland and the UK as one region for the purpose of sales. Thus, the same local vendors sell to both jurisdictions. Students typically use Braillenote Touch/Apex note takers (produced by Humanware) or similar products such as the BrailleSense produced by Hims Inc. These devices are often connected to monitors thereby enabling the teacher to see what the student is typing. As the newer note takers run on the Android platform, the range of applications available to support the student is more widespread than previously when older variants of the Windows CE operating system, with bespoke suites of applications, were the only things available on these note takers. One of the attractions offered by the Braillenote or BrailleSense families, is the presence of on board UEB converters. This means that, at least at theoretical standpoint, the student can enter mathematics in Braille, and the teacher can readily access it in printed notation. However, given the highly context-sensitive nature of Braille, and the inherent problems translating from context-sensitive grammars to those which are context-free, there will always be errors in the translation process. Whilst these can be minimised, it can be a concern that something entered correctly by the student in Braille is not translated accurately to the printed notation. This is particularly important in the case of examinations.

For those who do not know Braille, the options are more limited. Laptops are used with the typical sreenreaders (JAWS/NVDA) which are employed by blind users throughout the world. Whilst both screenreaders now have limited support for mathematical content, it is insufficient to enable a student to complete mathematics at the higher-level. The verbatim response of a visiting teacher is included here to highlight the issues:

"…I have a student who will sit her Leaving Certificate next year. She has only recently, and traumatically, gone blind. She does not know Braille, has no time to learn it as she is just about to finish her education. We have been able to provide her with text copies of all her other subjects apart from maths onto her laptop. She is a JAWS user and is an excellent typist. To assist with mathematics I have had to take her regular maths book (Texts and Tests 3 [24]) and transcribe it into a format which can be picked up by JAWS. Although JAWS and other screen readers can call out a range of mathematical symbols, it cannot do the complete list required by a Leaving Certificate student."

In order to combat this, the teacher has devised her own notation to represent the material. It is interesting that a parallel process of evolution has been followed here and in the Netherlands where a notation was devised. When questioned, the teacher who devised the notation was unaware of AsciiMath [1] and felt that the need to devise a bespoke notation for this student was present. The nature of the Irish context means that older and more traditional equipment is still very much used by blind students. The Perkins Braille Machine is, perhaps, the most common tool used to solve mathematical equations. This could be down to a lack of familiarity on the part of the students with the more complex devices such as Braille Notetakers, or the fact that the use of paper affords a two-dimensional interaction with the material; something which is lacking on other devices.

2.5 Access to Mathematics for Vision-Impaired students in Ireland.

The techniques and methods used by those with residual vision are similar in nature to those described above. As this demographic can access the printed form of mathematics (albeit in a slightly modified form) the range of tools available is arguably greater. Laptops are employed to read mathematics. Students utilise the usual magnification software which will enlarge the content. Word, and its in-built equation editor, is used by students to read material, and also to present answers to teachers. Most symbols have shortcut keys, which ensures that those who become familiar with their use can rapidly enter the information. It should be pointed out, however, that reliance on these shortcuts places an extremely high cognitive load on the user. This means that mental resources which are traditionally used to understand the material, are diverted to the task of entering it. This, as is well-known in the field of Human Computer Interaction, can induce fatigue and frustration on the user.

Tablets and smartphones are playing an increasingly important part in the educational opportunities for students with a vision-impairment. For example, it is now quite feasible to take a photograph of either the material presented by the teacher, and to have those snapshots available for perusal and study in the students' own time. Such applications as Graphical Calculators are becoming an integral part of the tools used by these students. As they are based primarily around apps, the normal Assistive Technology used by the students to provide generalized access to the tablet can also be used to access these tools. Interestingly, there are now apps which these students use to capture their handwriting, and which then subsequently turn this into text. Regrettably, the respondent who volunteered this information lacked sufficient knowledge to indicate precisely which apps are utilized, or how effective they are in the mathematical context. Again, quoting directly from the information provided by one respondent:

"As per the laptop, there are numerous sites where students can gain information regarding topics, find Apps relevant to inputting mathematical equations, etc. Mathletics, Khan academy, GeoGebra Classic, Quick Graph, GCSE Maths, Mathway, …these are some of the products which have been used by my students. The variety of Apps is enormous on the App store."

Revision of a topic can also be undertaken through the internet/google/Firefox/YouTube, etc. There are numerous sites which assist with mathematical learning on the internet. Accessing Google classroom is also quite common. Many students with a vision-impairment utilise services such as Edmodo. These sites offer digitally-based interactive classrooms and the fact that all material is provided digitally enables those who have some residual vision to interact with them.

As in the case of the blind students, some topics do not require a technological solution. Often concrete materials are a better option when trying to explain things. If the student is totally blind, and quite young, then concrete materials would have more relevance than a high tech solution. For vision-impaired students an enlarged copy of the topic, with relevant font sizes and styles, appropriately colour coded may be more important than any technology. Often vision-impaired students prefer to simply handwrite their answers into a standard or enlarged copy book. They may use adapted or standard graph paper. This information is included here to once again highlight the fact that though technology is becoming commonplace, it has not replaced the more traditional forms of interaction with this content.

2.6 Access to Diagrams for Blind and Vision-Impaired Students

In the case of both Ireland and the UK, there is nothing unusual in the methods used by both blind and vision-impaired students to explore diagrams. Many schools do have access to the technologies (such as Xyfuse and Piaf) to produce the raised-line images required by blind students. However, it has not been possible to determine whether any of the teachers, or other Special Needs Assistants have the requisite training to produce diagrams in an accessible format. In 2017, the Reading Services department of Childvision [7] began a pilot project to produce representations of 3D objects using 3D printers. As is pointed out in articles such as [25]blind children in particular prefer to have access to the object itself, rather than the diagrammatic representation thereof. Though no official studies have been carried out into the efficacy of this new initiative, informal feedback from teachers and students alike has been very positive. It is not known whether a similar initiative has been attempted in the UK, or whether 3D printing has been used to provide access to 3D objects in the same manner as has been done in Ireland.

According to information gleaned from teachers in the Special School for blind children, the most normal way of exploring visual artefacts is to use home-made models.

“The teacher who used to be in charge of the maths department has retired” said one teacher, “and he left a lot of bits and pieces behind. He used the inserts from toilet roles, tins covered in paper, and lots of other things that would otherwise have been thrown out. He built up an enormous collection over the years”. The importance of experiential knowledge gained by teachers cannot be over-stated. In the case of the information provided above, the teacher who constructed the artefacts had been working with blind and vision-impaired students for over forty years. As well as these home-made models, it is commonplace to utilise rubber mats and thin sheets of plastic foil to depict diagrams such as graphs etc. The reader may not be aware that, when thin sheets of paper are placed on soft rubberised surfaces, and an image is drawn on the plastic, it appears as a raised-line mirror image on the reverse of the plastic sheet. This crude, though effective means of conveying some diagrams to blind students has been used in Ireland and the UK for many years.

The reason for the prevalence of this seemingly primitive method of preparing diagrams in this way is twofold. Firstly, it is only in recent years that technology has started to play a significant role in the education of any children in Ireland and the UK. Also, many teachers simply find it more expedient to sketch a rough outline of a graph using a rubber mat and plastic foil, than to use complex hardware and software to achieve more or less the same result. This viewpoint very much echoes the sentiments found in [25].

For those students with residual vision, it is arguably easier to access diagrammatic material. For many, the material is presented by either magnifying the image on a tablet or laptop. This, however, induces similar problems to those faced by blind students when exploring tactually. When the diagram is magnified on-screen, only a small part is visible at any given moment. Thus, it can be extremely difficult for students with a vision-impairment to gain an overview of the entire image. What many prefer to do in order to combat this is simply to print the graphic on a larger piece of paper. Using this slightly cruder interaction, they can more readily use their remaining vision to scan over the enlarged picture to gain the necessary insights. It should also be noted that larger graph-paper is also used for the same reasons, and in the same context

2.7 Discussion

The nature of the Irish context is such that a mixture of traditional, and technological methods is employed to ensure that blind/vision-impaired students have access to mathematics. It is a matter of concern, as highlighted in [26]that the numbers of students with a visual disability are not attending university, and other third-level courses in the same numbers as students from other demographics. The information presented in the previous sections has significant implications for the further development of the PlatMat suite of applications.

Firstly, in order to ensure that the tools are usable in Ireland, the ability to produce accurate Braille representations in UEB [23] and furthermore, that it be possible to produce paper-based representations in this format. Whilst no formal evaluation of the PlatMat tools has been undertaken in Ireland as yet, the Reading Services team [7]were particularly enthused by the choice of EPUB3 as the representation. They felt that, if the data-entry aspects of the material could be made rapid, that further development would give them a viable alternative to the process described in Section 2.4 which is currently used for production of textbooks.

As was noted in section 2.4, the use of electronic Braille devices and laptops is becoming more and more prevalent. Thus, as the example provided by the Visiting Teacher showed, it is imperative that any tools cater for students who prefer this modality. Two alternatives present themselves at this point. Should the newly developed tools cater for this innovative notation devised by the Visiting Teacher to support her/his student, or should training be given in a standardised notation such as AsciiMath [1] and thereby open the opportunities to use alternative software which use this notation. Further investigation will need to be undertaken to determine how widely this newly devised notation is being used. If it has become a de facto standard amongst those who support the education of blind/vision-impaired students, then the tools developed as part of the EuroMath project must cater for it. If, on the other hand, it is only being used by one individual, then perhaps part of the outputs of the project will be training materials to assist teachers in Ireland to use notations, and also other software, that are in common usage in other countries.

The education of blind/vision-impaired students in Ireland in the field of mathematics is very much at a crossroads where old meets new. EuroMath, based on the previously implemented PlatMat tools, is ideally placed to influence the direction which it may take. Not only will the tools be useful for the students and their teachers alike, but the 300 examples highlight best-practice across the domain will prove invaluable. Indeed, when speaking with the teachers in the Special School, it was the proposed sharing of this content that created the most excitement. Whether this is down to a lack of knowledge concerning what can be achieved through the use of technological solutions, is not known. What can be said is that it is imperative that projects such as EuroMath work to give students in Ireland greater access to mathematics, and thereby ensure that they have a far greater chance of attaining both academic and professional success.

3 Teaching Mathematics to Blind and Visually Impaired Students: The Dutch Context

3.1 Education in the Netherlands

3.1.1 Introduction to the education system

Although the mandatory age for school attendance in the Netherlands is five, almost every child starts in primary school as soon as possible after his or her fourth birthday. The actual learning begins in the child's third year. Depending on the school, subjects such as natural sciences, geography, history and even English are taught. However, the emphasis is on reading, writing and arithmetic [27].

By the end of primary school, at the age of about twelve (see Figure 2) children must take a compulsory exam, the cito test. Based on the results of this exam, teachers recommend the level of secondary education that is most suitable for the child. Children who get a cito score high enough to go to VWO (pre-university secondary education) or HAVO (senior general secondary education) will almost certainly continue in higher education, while more practically oriented children go to VMBO (pre-vocational secondary education) for vocational training. When the child attends VWO, he or she will be in school for six years and will do his or her final exams when about eighteen years old. Upon passing the VWO exams he or she can be admitted to a university. The HAVO curriculum lasts five years and paves the way to higher vocational education at an HBO (higher professional education) college. There are four different programs for VMBO students, varying from the very practical to more theoretical training. After four years, the student can continue their education at an MBO (secondary vocational education) school. Children can move from one level to another in the course of their school career, based on their results. So a VMBO student with straight 'A's can go on at HAVO level, and a struggling' VWO pupil can go back to HAVO level [27].

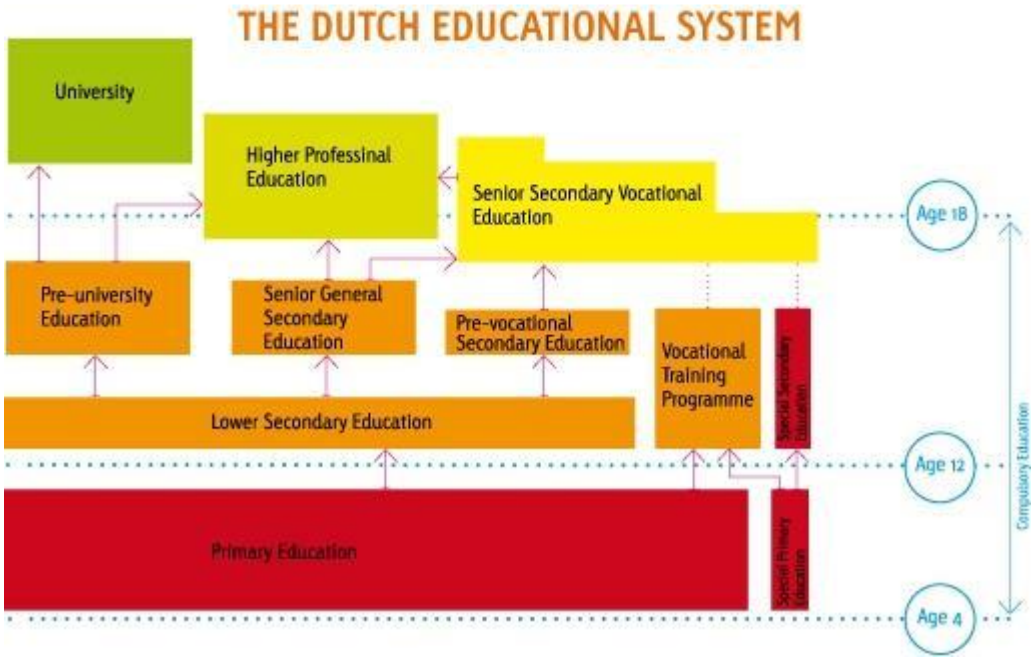


Figure 2 The Dutch Education System Retrieved 11th April 2018 [28]

3.1.2 Teacher training

If an individual desires to become a primary school teacher they must study at a higher professional education institution (so called “PABO”). The PABO-students are trained to teach all educational programs in primary schools. The initial teacher training program includes an introduction to the training of pupils with special needs. Qualified primary school teachers are eligible to teach every subject at primary and secondary level in special education [29].

As far as teaching in secondary regular education is concerned, there are two forms of educational qualification:

- Lower secondary qualification: this so-called 'grade two' qualification qualifies teachers for teaching the first three years of HAVO and VWO and all years of secondary vocational education (VMBO/MBO). In general, teachers are qualified to teach one specific subject. Courses for this level are given in higher professional education institutions (the previously mentioned HBO colleges).
- Full qualification: this 'grade one' qualification qualifies teachers for all levels to teach in secondary education. In general, teachers are qualified for one specific subject. The first qualification courses are given to institutions for higher professional education and universities.

3.1.3 Different mathematical packages in HAVO and VWO

At VMBO there are 3 programmes, each programme offers math education on a different level, within these levels all students take the same exams. This is different for HAVO and VWO students. At the end of the third year of secondary school the students choose their HAVO and VWO subjects for final examination purposes. They have to choose one of four so-called study profiles with a unique combination of subjects each. These profiles are introduced to improve with reference to further education. The four study profiles are as follows: science & technology (abbreviated as NT), science & health (NG), economy & society (EM), and culture & society (CM).

VWO offers four mathematical packages: mathematics A (light), mathematics B, mathematics C (ultra-light) and mathematics D (extra). HAVO offers the same packages except for mathematics C. Mathematical D is an optional subject which is not required for any form of further education. A student can follow mathematics D only in combination with mathematics B, and only in the science & technology profile.

Every profile has its own mathematical package, except for the CM profile at HAVO level. In this profile mathematics is not required. It should be clear that mathematics B, which is the most difficult package besides mathematics D, is required in the science & technology profile.

Inspection of Figure 3 shows that in 2016 more than 80% of the VMBO students and about 95% of the HAVO students did their final exams in mathematics. Inspection of Figure 4 shows that, in 2016, more than 25% of the HAVO students and almost 50% of the VWO students did their final exams in mathematics B

.

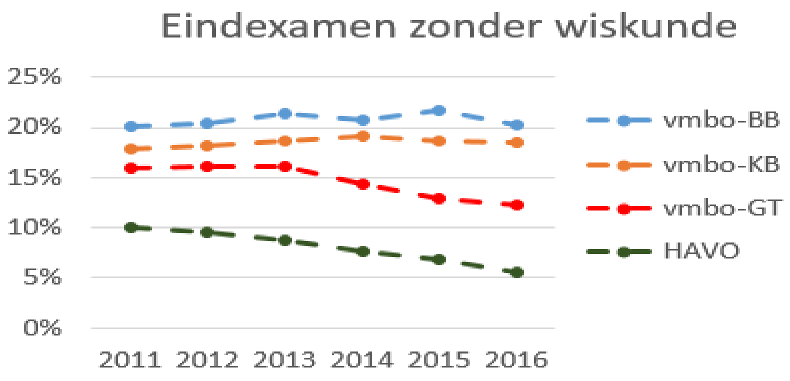


Figure 3 Final Exam Without Mathematics (retrieved from (http://www.wiskundebrief.nl/762.htm)

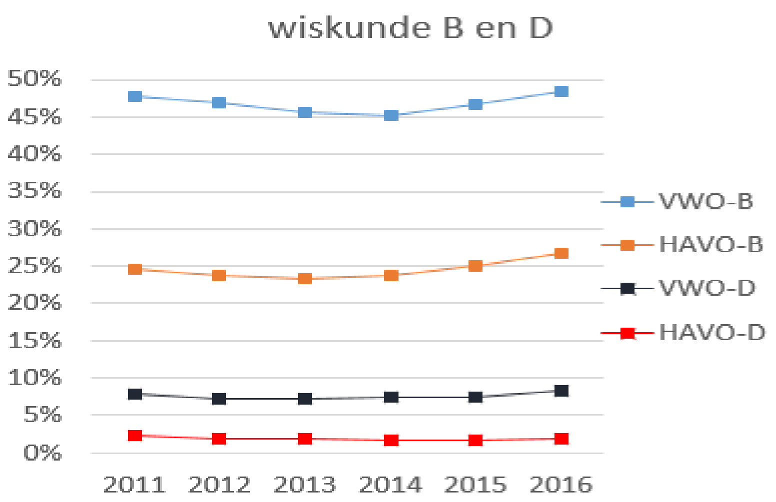


Figure 4 Math B and D (retrieved from (http://www.wiskundebrief.nl/762.htm)

(Section 3.8.1 includes the whole program of the VWO mathematics A, B and C).

3.2 Education for Blind and Visually Impaired Students

3.2.1 Introduction

Visio Education (www.visio.org) and Bartiméus Education (www.bartimeus.nl) provide education for children who are visually impaired or blind. This takes place at their own schools, the schools for blind and visually impaired students, and by guiding blind and visually impaired students in regular education. Through the use of itinerant education support in regular schools, Visio Education and Bartiméus Education make it possible for a visually impaired or blind student to attend a mainstream school in a familiar environment.

3.2.2 Itinerant education support

Itinerant educational support is available for pupils in primary school, in special education, in all types of secondary education, vocational education (MBO), and if necessary at the beginning of the higher vocational education (HBO) or the University. Approximately 89% of the visually impaired students and 47% of the blind students in primary education go to a mainstream school. About 79% of the visually impaired students and about 36% of the blind students in secondary education go to a mainstream school [30]. In mainstream education, students who are visually impaired or blind are guided by itinerant teachers who are, almost always, qualified primary school teachers. The itinerant teacher helps students with visual impairments to participate to the best possible degree in the mainstream educational setting. The itinerant teacher can offer advice to teachers on a suitable learning environment and on adjustments of the lesson material. He or she not only advises on certain tools, but also provides support in how to use them [31]).

3.2.3 Special education

Primary special education for blind and visually impaired students

If a child is visually impaired or blind and cannot meet the requirements of the regular school, or if the school cannot offer an optimal adaptive learning environment, then this child can participate in a school for special education. The teachers are committed to achieving the same core objectives as in mainstream schools, however they work with components that are adapted or supplemented because of the students' visual disability. Children develop the same knowledge and skills as their peers in the mainstream setting. Since it is essential that everyone is able to follow the lessons, the tempo is usually lower and the groups are small: six to twelve pupils per class. Although a school for blind and visually impaired students is, in some ways, a typical school, a lot of adjustments are required because of the visual disabilities of the students: the design of the school and the classroom, the selection of methods and (adapted) materials, the use of assistive devices and the didactical and pedagogical approach. The teachers attempt to make the visible world understandable for blind and visually impaired students. In addition to their visual disability, many of the students in special education also have other problems, such as socio-emotional problems and learning problems [32].

Secondary special education for visually impaired students

Blind and visually impaired students who can no longer keep up with the demands of the mainstream secondary schools, may attend special educational establishments. At these schools, the student may avail of education at all levels, except the VWO-level. The requirements and contents are the same as in the mainstream secondary schools, however the tempo for completing the programme of study is somewhat slower. The teachers offer tailor-made programs to the students and the teaching is adapted to the disabilities of the students. There are adaptations in the materials and lesson methods. Many students use assistive technology devices [32].

3.2.4 Support for math teachers teaching blind and visually impaired students in Mainstream secondary education

Math teachers often only feel slightly confident in their ability to provide adequate educational support to visually impaired students. This is especially the case when the students are blind. Therefore, the blind students and their teachers get support. The blind student receives a math box with various kinds of materials which can be used in math lessons (see Figure 5). For example, a tactile ruler and a tactile coordinate plane. Furthermore, Visio and Bartiméus Education offer one-day courses to math teachers of blind students.

Figure 5 Mathbox for Blind Students in Mainstream Secondary Education (retrieved from https://www.eduvip.nl/wiskundekist-voor-braille-en-zeer-slechtziende-leerlingen/)

In these face-to-face courses, teachers may exchange experiences and knowledge with each other. They learn and practice the mathematical notation for blind students. The teachers imitate reading on the braille display (see Figure 6) as well as reading with the speech synthesizer which gives them a lot of insight into the challenges that blind students encounter while reading and comprehending mathematical expressions and equations. The teachers also learn how to teach blind students to read tactile and audible graphs.

Figure 6 Braille Display Connected to a Laptop (retrieved from http://www.hims-inc.com/product/braille-edge-40/)

In addition to these courses, teachers can get information on EduVip [33]. This is a website developed by Visio and Bartiméus Education that provides a lot of information on the education of blind and visually impaired students.

Finally, math teachers and blind and visually impaired students can get individual support in mathematics, if needed, from a professional at Visio Education or Bartiméus Education.

3.3 Research on assistive technology for visually impaired students

3.3.1 Research 2016 (Netherlands)

In May 2016 Visio Education carried out research on how primary and secondary students who were blind or visually impaired used their assistive devices. 44 students, attending a school for blind and visually impaired students, from age 10 to 20, participated in this study. They had to complete a questionnaire. Furthermore, the researchers received information from visiting teachers about how blind and visually impaired students used their devices in regular education. [34]

At the special school 100% of the blind and visually impaired students who were older than 10 years of age, used a laptop. The blind students worked on a laptop with a braille display and speech synthesizer. All blind students were initially introduced to training in the skills needed to type. After finishing this training, they began a programme in basic ICT literacy. At the commencement of this course, the children were about 8 to 10 years old. They were taught to use the laptop with a braille display and speech synthesizer in Word, Outlook, Windows and on the Internet. They were provided with individual training for half an hour per week, which lasted for approximately two years. 95% of the blind students used the screen reader Jaws, 5% used NVDA. Out of the visually impaired students 15% used Window magnification, 32% Supernova magnification, 8% Supernova magnification plus speech synthesizer and 45% used no additional devices. 25% of the secondary students at this school used the smartphone as a daisy player [35]

In mainstream education, the blind students started to work on a laptop with a braille display at the age of 5 to 6 years old. They were trained by the itinerant teacher. 95% of the partially sighted students, from the age of 7, used a laptop. 5% used a tablet: OS or Android. 35% of the visually impaired students used a laptop plus tablet. Many of this demographic used a board camera or screen sharing software which transmits content on the whiteboard from the teacher's computer directly to the students' laptop or tablet via internet of Wi-Fi (e.g. Join.me, TeamViewer, skype) [36]

Dedicon (www.dedicon.nl), the Dutch non-profit organization that makes information accessible to people with a print disability, delivers documents in MS Word-format (so-called Edu-format), in PDF-format (e.g. pixel-based, vector-based, black and white, colour, protected environment) and in Daisy-format (e.g. Lex). Dedicon use headings and page numbering (which correspond directly with the printed book used by sighted classmates) in their Word documents. These adaptations enable a fast and easy navigation in a Word document, with Jaws or another screen reader. Visually Impaired students often use the pdf exchange editor. This editor enables writing in PDF. In addition, students have at their disposal an arsenal of tools, such as rulers and grids.

3.3.2 Research 2018 (Netherlands)

For the purpose of this report, in 2018 a questionnaire was sent to itinerant teachers of blind and visually impaired students in mainstream secondary education and asked them to forward these questionnaires to their teachers of mathematics. 29 of those who teach visually impaired secondary students and 9 who teach blind secondary students filled in these questionnaires.

In Table 1 the results of the teachers of the partially sighted students are shown. 72% of all teachers used the interactive whiteboard in their math lessons. 52% of the visually impaired students used a laptop. 58% of the visually impaired students magnified text, graphs and drawings. 21% used more contrast for text, on the laptop. 24% more contrast for graphs and drawings. One visually impaired student used the speech synthesizer. There were four visually impaired students (14%) who used the touch screen.

	Yes	No	Other
1 Do you use an interactive white board in Math Class?	21	5	3
2 Which of the following devices do the sighted students use in your mathematics lesson?			
2a laptop	10	18	1
2b tablet	11	18	0
2c smartphone	7	19	3
3 Which of the following devices does the visually impaired student use in your mathematics lesson?			
3a laptop	15	13	1
3b tablet	7	21	1
3c smartphone	7	17	5
4 How are the tools used by the visually impaired student?			
4a The text is enlarged on the screen.	17	7	5
4b Graphs and drawings are enlarged on the screen.	17	7	5
4c The text is displayed with more contrast on the screen.	6	16	7
4d Graphs and drawings are shown on the screen with more contrast	7	14	8
4[e]			
4f The text is converted into speech using speech synthesis software	1	29	0

4g The partially sighted student uses a touch screen for solving mathematics assignments	4	25	0

Table 1 Results of questionnaires completed by 29 math teachers of visually impaired students in secondary schools (mainstream) in the Netherlands

	Yes	No	Other
1 Do you use an interactive white board in Math Class?	6		1
2 Which of the following devices do the sighted students use in your mathematics lesson?			
2a laptop	1	5	1
2b tablet	1	5	1
2c smartphone	1	3	3
3 Which of the following devices does the blind student use in your mathematics lesson?			
3a laptop	7	0	0
3b tablet	0	7	0
3c smartphone	1	6	0
4 How are the tools used by the blind student?			
4a not applicable for blind students			
4b not applicable for blind students			
4c not applicable for blind students			
4d not applicable for blind students			
4e The text is displayed in braille	7	0	0
4f The text is converted into speech using speech synthesis software	4	3	0
4g The blind student uses a touch screen for solving mathematics assignments	0	7	0

Table 2 Results of questionnaires completed by 9 math teachers of blind students in secondary schools (mainstream) in the Netherlands

In Table 2 the results of the math teachers of blind students are shown. 86% of the math teachers (6 out of 7) used an interactive whiteboard. All blind students used a laptop while doing mathematics. All students used the braille display. 57% of the blind students used a braille display and a speech synthesizer. No blind students used the touch screen.

Finally, in 2018 Visio Education developed roadmap for deployment of software. They stated that it is important that:

- software should be platform independent, e.g. BYOD, iOS, Windows and Android
- software and digital content, including the educational websites, should be 100% accessible
- software language can be changed independently of the operating system language (independent language stetting).
- students, at all times, can use their own magnification software and software speech synthesizer
- (if possible) free software is used

3.3.3 Research 2018 (Belgium)

In 2018 we sent the same questionnaire to itinerant teachers of blind and visually impaired students in mainstream secondary education in Belgium (Spermalie, Brugge) and asked them to forward these questionnaires to their mathematical teachers. 7 mathematical teachers of visually impaired secondary students and 1 mathematical teacher of a blind secondary student filled in these questionnaires.

In Table 3 the results of the teachers of the visually impaired students are shown. 43% of all teachers used the interactive whiteboard in their math lessons. 29% of the visually impaired students used a laptop. 57% of the visually impaired students magnified text, graphs and drawings. 17% used more contrast for text, graphs and drawings. No visually impaired student used the speech synthesizer. One visually impaired student used the touch screen. One math teacher of one blind student filled in the questionnaire. See Table 4. The student used a laptop with a braille display and speech synthesizer while doing mathematics.

	Yes	No	Other
1 Do you use an interactive white board in Math Class?	3	4	0
2 Which of the following devices do the sighted students use in your mathematics lesson?			
2a laptop	2	5	0
2b tablet	1	6	0
2c smartphone	1	5	1
3 Which of the following devices does the visually impaired student use in your mathematics lesson?			
3a laptop	2	4	1
3b tablet	1	6	0
3c smartphone	2	5	0
4 How are the tools used by the visually impaired student?			
4a The text is enlarged on the screen.	4	2	1
4b Graphs and drawings are enlarged on the screen.	4	2	1
4c The text is displayed with more contrast on the screen.	1	6	0
4d Graphs and drawings are shown on the screen with more contrast	1	6	0
4[e]			
4f The text is converted into speech using speech synthesis software	0	6	1
4g The visually impaired student uses a touch screen for solving mathematics assignments	1	6	0

Table 3 Results of questionnaires completed by 7 math teachers of partially sighted students in secondary education in Belgium.

	Yes	No	Other
1 Do you use an interactive white board in Math Class?		1	
2 Which of the following devices do the sighted students use in your mathematics lesson?		1	
2a laptop		1	
2b tablet		1	
2c smartphone		1	
3 Which of the following devices does the blind student use in your mathematics lesson?			
3a laptop	1		
3b tablet		1	
3c smartphone		1	
4 How are the tools used by the blind student?			
4a not applicable for blind students			
4b not applicable for blind students			
4c not applicable for blind students			
4d not applicable for blind students			
4e The text is displayed in braille	1		
4f The text is converted into speech using speech synthesis software	1		
4g The blind student uses a touch screen for solving mathematics assignments		1	

Table 4 Results of a questionnaire completed by a math teacher of a blind secondary student in Belgium

3.4 Access to Mathematical Expressions and Equations for Blind and Visually Impaired students

3.4.1 Introduction

In the Netherlands blind students in secondary education rarely read math from printed braille. They work on a laptop with screen reader software. This software attempts to convey what sighted people see on the display to them via braille or synthetic speech and, thus, enables blind students to read and comprehend mathematical text.

3.4.2 Math books and mathematical notation

Math books in the Netherlands are made accessible by Dedicon. All blind students, in secondary education, use books in Word format. Drawings and graphs may be provided in tactile format. E-learning products, used by the sighted classmates, are often not accessible for blind students. The graphic calculator which sighted students have to use in upper secondary education in HAVO and VWO, is also not accessible for blind students. As an alternative, the blind students use Excel in combination with AllerCalc.

The blind students read the (mathematical) text, typed in a Word-format, on the braille display and/or with the speech synthesizer. The teacher (or sighted classmate) can read the “same” (mathematical) text on the screen of the laptop and does not have to learn braille. The mathematical notation that is depicted on the screen is a linear notation, quite similar to Excel (see Table 5). This notation only uses keys that are on the QWERTY keyboard. The display of the text is “one-to-one” on an 8 dot braille display.

Notation for Sighted Students	**Notation, in Word-format, for blind students**
x^2	x^2
$\sqrt{16-4x}$	sqrt(16 - 4x)
$sin(\propto + \beta)$	sin(~a + ~b)
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	x = (-b +- sqrt(b^2 - 4ac))/(2a)
$1 - \sqrt{\frac{x-2}{x^2-4}} = 0$	1 - sqrt((x - 2)/(x^2 - 4)) = 0

Table 5 Mathematical notation for sighted and blind students

3.4.3 Mathematical expressions and equations

The presentation of mathematical expressions and equations can present difficulties because these representations are poor in providing context, very compact and non-linear in nature. Confusion with braille notation and insufficient knowledge and skills for using their braille display and synthetic speech contributes to the challenges experienced by blind students when engaging with expressions and equations.

In November and December 2017, Visio Education carried out a pilot with three blind students who performed, best in mathematics in mainstream secondary education out of all blind students in the Netherlands [37]. They examined how these blind students read and comprehend mathematical expressions and equations using a braille display and the speech synthesizer. The students had a lot of difficulties with decoding, reading and comprehension of the expressions and equations in braille. They also used the speech synthesizer, however to nowhere near the same extent. For example, mathematical symbols such as “^” and “*” were not pronounced. The students were not taught how to use these devices while doing mathematics. According to the three students, the advantage of using the braille display compared the speech synthesizer is that you have more control, you see more detail and more structure and you are less isolated from your peers. The advantage of the speech synthesizer compared the braille display is rate of reading.

Most blind students use the braille display as well as the speech synthesizer while solving mathematical expressions and equations. Some students use hot keys like paste, copy, etcetera to save time while solving mathematical expressions and equations. Most blind students have learned to type as few intermediate steps as possible. This is a pity because typing intermediate steps can relieve their working memory. In addition, in secondary education, students receive a lower grade if they do not show how they solved the problem.

In general, it is very helpful when students can make their own products while doing mathematics. An example of such a product is depicted in Figure 7. It is a sketch made by a sighted student. The sketch helped her to calculate 724 – 356 = . Blind students can't make these kinds of drawings.

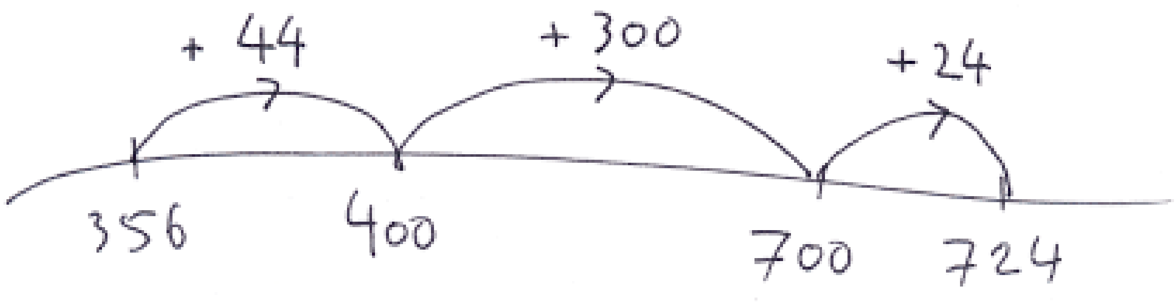


Figure 7 Own production of a sighted student.

Visually Impaired students can read and write in different formats such as from PDF, Word or on paper. They may choose to use the regular mathematical notation, or the linear representation (i.e., that used by blind students).

3.5 Access to Graphs for Blind and Visually Impaired Students in the Netherlands

3.5.1 Introduction

The relationship between two variables is often depicted in a graph. Graphs can be represented in an audio or tactile format. An audible graph is very suitable for giving a global picture of the relationship between two variables in less than a second. A tactile graph is more suitable for getting information on a more detailed level. The relationship between two variables can also be represented in a table. Teachers have to teach blind students how to use and select the different representations. "Reading" and making tactile graphs is very time consuming. At a certain point, blind students can learn to mentally represent graphs to save time. This will be explained in the text below.

3.5.2 Another way of looking at graphs

In mathematics, there are different kinds of graphs that one may see more often than others. Each has its own type of function that produces the graph. In this document, the discussion will only focus on the graphs of linear functions and those based on of quadratic functions.

The graph of the function y = x is a straight line passing through the origin with a slope of 1. Braille readers have to make this graph on a tactile coordinate plane and study the special features of this graph. By transforming the function, the graph can be translated, reflected or otherwise changed. Take as an example the function y = x + 3. The graph of this function can be obtained from the graph of y = x by translating 3 units along the y-axis. So, if you learn what the graph of y = x looks like, it is easy to imagine how the graph of y = x + 3 appears. You do not need to make this graph.

Let us describe another example. The graph of y = x^2 has the form of a so called parabola. This parabola opens up. Braille readers have to make this graph on a tactile coordinate plane and study the characteristics of this graph (e.g. the graph is symmetric, it opens up). The graph of y = x^2 + 2x – 8 can be obtained from the parabola y = x^2 by translating -1 units along the x-axis and -9 units along the y-axis. This is because the equation is equal to y + 9 = (x + 1)^2 (see Figure 8). Again, students do not need to make this graph, they can imagine what it looks like.

These two examples show how graphs change by transformation of the functions. When students understand this, they no longer need to make “all” the graphs. They can give a precise description of the form graph and describe the construction of the graph.

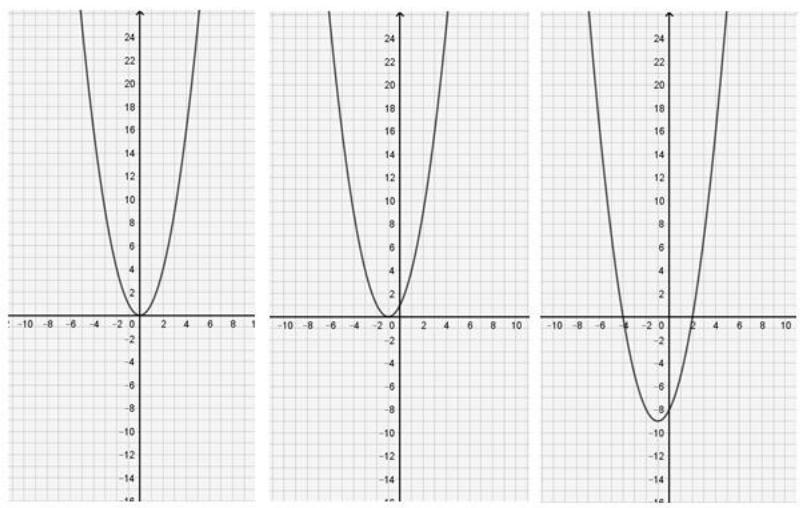


Figure 8 Different parabolas, all with the basic form of y = x^2

There are also some interesting software programs which enable students to read and construct graphs. In the Netherlands, the most used programs for sighted students are Desmos (https://www.desmos.com/) and GeoGebra (https://www.geogebra.org/). Desmos is an advanced graphic calculator and is increasingly popular in regular secondary education.

This calculator is accessible for blind students. GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics from primary school to university level. This application is used a lot by sighted students and their mathematical teachers in secondary education. It is accessible for most visually impaired students, but not for blind students.

3.6 Examinations

3.6.1 Examinations for mainstream secondary education

The school-leaving examination for secondary education involves a school examination and a national written examination at the end of the final school year.

School examination

Schools compose their own intermediate exams, however the Ministry of Education, Culture and Science dictates which subjects must be taught during the exam year. These school examination dates are not nationally determined: schools are free to test pupils in specific subjects whenever they wish. The school exam usually consists of two or more tests per subject, which may be oral, practical or written. Subjects outside the national exam framework may therefore be completed before the final year of school.

National examination

There is one national written exam per subject for all pupils who receive the same type of education. Whether a subject is mandatory or optional, the exam questions are the same across the country. The national exam always takes place at the end of the final year and is constructed by the Ministry of Education, Culture and Science.

Blind and visually impaired students who attend a mainstream school must take the intermediate school exams as well as the national examinations. The school can adapt their intermediate school examinations for blind students. The national mathematical examinations are adapted by four qualified math teachers who have a lot of experience in teaching mathematics to blind and visually impaired students. The aim is to offer equitable exams to blind students. In Table 6 some examples of adaptations and support for visually impaired and blind students are provided. The adapted examinations arrangement for the national exams will be discussed with the individual student. The adapted examinations from the previous few years, can be found on www.eduvip.nl. With respect to adaptations for blind students, "everything is possible" as long as the examinations are equitable to the regular examinations (see Table 6).

Adaptation or support	Sighted student	Visually Impaired student	Braille student
Extra time?	No	Yes, 50%	Yes, 100%
Format of exam	(in general) paper	(enlarged) paper, PDF-file, Word-(Edu-) file, Daisy	Word-(Edu-) file
Mathematical notation	”regular” notation	regular notation or linear notation (.i.e. the notation for blind students)	linear notation (accessible for blind students)
Graphs	depicted on paper	depicted on paper or on the screen of the laptop	a description of a graph and/or a tactile graph
Drawings	depicted on paper	depicted on paper or on the screen of the laptop	a description of a graph and/or a tactile drawing and/or a model of the drawing. The tactile drawing is often a simplification of the original drawing
Adaptation of content?	No	No	If necessary and desired: yes. The aim is an equitable exam
A supervisor for help with reading graphs, drawings or tables	No	Yes	Yes

Table 6 Examples of adapted examinations arrangements for blind and visually impaired students

3.6.2 Examinations for special secondary education

Students who attend the school for blind and visually impaired students have to take the same (adapted) national examinations. Instead of school examinations they have to take oral examinations. These examinations are held by an independent commission.

(In Section 3.8.2 you will find an adapted task of the VMBO-GL and TL examination, 2017)

3.6.3 Examination in math B

Less than 10% of blind students did their final examination in mathematics B in the last 10 years [38]. Figure 4 shows that about 25% of the sighted students at HAVO level and about 50% of the sighted students at VWO level did their final examination in mathematics B.

3.7 Discussion and commentary

For the purpose of this project research 36 math teachers of partially sighted students, 29 from the Netherlands and 7 from Belgium, participated in this research (in 2018). 8 math teachers of blind students, 7 from the Netherlands and 1 from Belgium, contributed to this research. They filled in a questionnaire (see Table 1 to Table 4).

All blind students, in Belgium and in the Netherland, used a laptop with a braille display. 57% of the blind students from the Netherlands and 1 blind student from Belgium used a braille display and a speech synthesizer while doing mathematics. None of the blind students used a touch screen.

52% of the visually impaired students from the Netherlands used a laptop; 29% from Belgium. Only one visually impaired student, (from the Netherlands) used the speech synthesizer. In the Netherlands, 4 visually impaired students (14%) used the touch screen, whereas in Belgium only one student used touch screen.

One problem is that blind students are not taught (well enough) how to use braille in conjunction with the speech synthesizer while doing mathematics. For example, they should learn how to filter information while reading equations and expressions on a braille display. They should learn how to get a global overview of the entire equation or expressions by using the speech synthesizer and/or the braille display. They also need more training in how to read and write the linear math notation. This is also important for visually impaired students who use this notation.

Furthermore, it is very important that students can make their own productions while solving mathematical expressions and equations (see, for example, Figure 7). At this moment, there are very few possibilities for blind students to work on their own productions. In the EuroMath project, we aim to find a solution for this problem.

In addition, blind students need to learn how to read and make audible and tactile graphs.

Also we aim to implement the recommendations outlined by the ideas of Visio Education on software:

- software should be platform independent, e.g. BYOD, IOS, Windows and Android
- software and digital content, including the educational websites, should be 100% accessible
- software language can be changed independently of the operating system language (independent language stetting).

- students, at all times, can use their own magnification software and software speech synthesizer
- (if possible) free software is used.

3.8 Supplemental Information

3.8.1 Appendix A

VWO mathematics A (retrieved from http://www.boswell-beta.nl/vwo/math-a)
Topics include: • Probability and chance • Statistics within the realm of the binomial distribution • Statistics within the realm of the hypergeometric distribution • Statistics within the realm of the normal distribution: mean, standard deviation, z-score, hypothesis statement, significance level, hypothesis testing (z-test) • Trigonometric functions: sine and cosine functions • Trigonometric equalities and inequalities: exact solutions • Arithmetic and geometric sequences and series • Sum totals within series • Exponential and logarithmic functions: equalities and inequalities; conversion of exponential to logarithmic, vice versa • Differential calculus: derivatives of power functions • Product rule, quotient rule and chain rule • Extremes and tangents, velocity
Requirements: graphing calculator, protractor

Table 7 The content of the Dutch VWO mathematical A examination

VWO mathematics B (retrieved from http://www.boswell-beta.nl/examen/vwo/mathematics-b)
Topics include: • Differentiation: power functions, exponential functions, logarithmic functions, trigonometric functions, product rule, quotient rule, chain rule, extremes and inflection points • Integration: power functions, exponential functions, logarithmic functions, trigonometric functions, area between functions, volume of revolution around X-axis or Y-axis, centre of mass, arc length. • Trigonometry: sum and difference formulas, Simpson formulas, trigonometric functions, harmonic functions Lissajous figures (parametric curves) Proofs in Euclidian Geometry: • Proofs of congruence and similarity of triangles • Perpendicular bisector and outer circle, theorem of perpendicular bisectors

• Bisector and inner circle, theorem of bisectors • Altitude line, median line, theorem of altitude lines, theorem of median lines • Quadrangles, quadrilaterals, theorem of quadrilaterals • Central angle, circumferential angle, theorem of circumferential angle • Theorem of Thales, theorem of a constant angle, theorem chord and tangent • Inverse theorem of Thales, inverse theorem of constant angle • Proofs from the absurd
Requirements: calculator, protractor, compass

Table 8 The content of the Dutch VWO mathematical B examination

VWO mathematics C (http://www.boswell-beta.nl/examen/vwo/mathematics-c)
Topics include: • Functions and graphs • Linear and quadratic functions • Exponential and logarithmic functions: equalities and inequalities; conversion of exponential to logarithmic, vice versa • Arithmetic and geometric sequences and series; sum totals within series • Descriptive statistics: summarising data • Systematic counting: factorial, permutation, combination • Probability and chance • Random variables, expectation, standard deviation • The binomial distribution • The hypergeometric distribution • The normal distribution: mean, standard deviation, square root N law • Normal approximation of the binomial distribution
Requirements: calculator; Protractor

Table 9 The content of the Dutch VWO mathematical C examination

3.8.2 Appendix B

Question 3 of examination VMBO-GL and TL 2017

This examination is adapted for blind students. We show here only the questions belonging to “Final height”. The exam was presented in a Word-format. The drawings were offered in raised-lines.

Question 3 was adapted. The original question was:

Take, for the height of the father, 180 cm. In the coordinate system on the extra paper, draw the graph that corresponds to the formula. You can use the table. Make yourself a correct distribution on the vertical axis.

Examination VMBO-GL and TL 2017 for blind students

Final height

If you know the height of a girl's father and the height of a girl's mother, you can calculate the expected final height of this girl with the formula:

final height = (height father + height mother - 13)/2 + 4,5

The final height, father's height and mother's height are expressed in centimetres.

Question 1: 2 points

The height of Nicolette's father is 185 cm and the height of her mother is 170 cm.

Calculate how many cm the expected final height of Nicolette is. Write down your calculation.

Question 2: 3 points

Carla no longer grows. Her final height is 190 cm. Her father is 2 meters long.

Calculate how many cm the height of Carla's mother should be according to the formula. Write down your calculation.

The average height of a Dutch man is 180 cm.

Question 3a: 2 points

Take, for the height of father, 180 cm. Copy and fill the following table

start table

column 1: height mother (cm)

column 2: final height (cm)

140; ...

150; ...

160; ...

170; ...

180; ...

190; ...

200; ...

end table

Question 3b: 2 points

See drawing 1.

In the coordinate system of drawing 1 a graph can be drawn which belongs to the formula.

Indicate the value at point A after the saw tooth and specify the step size on the vertical axis so that the graph can be displayed correctly.

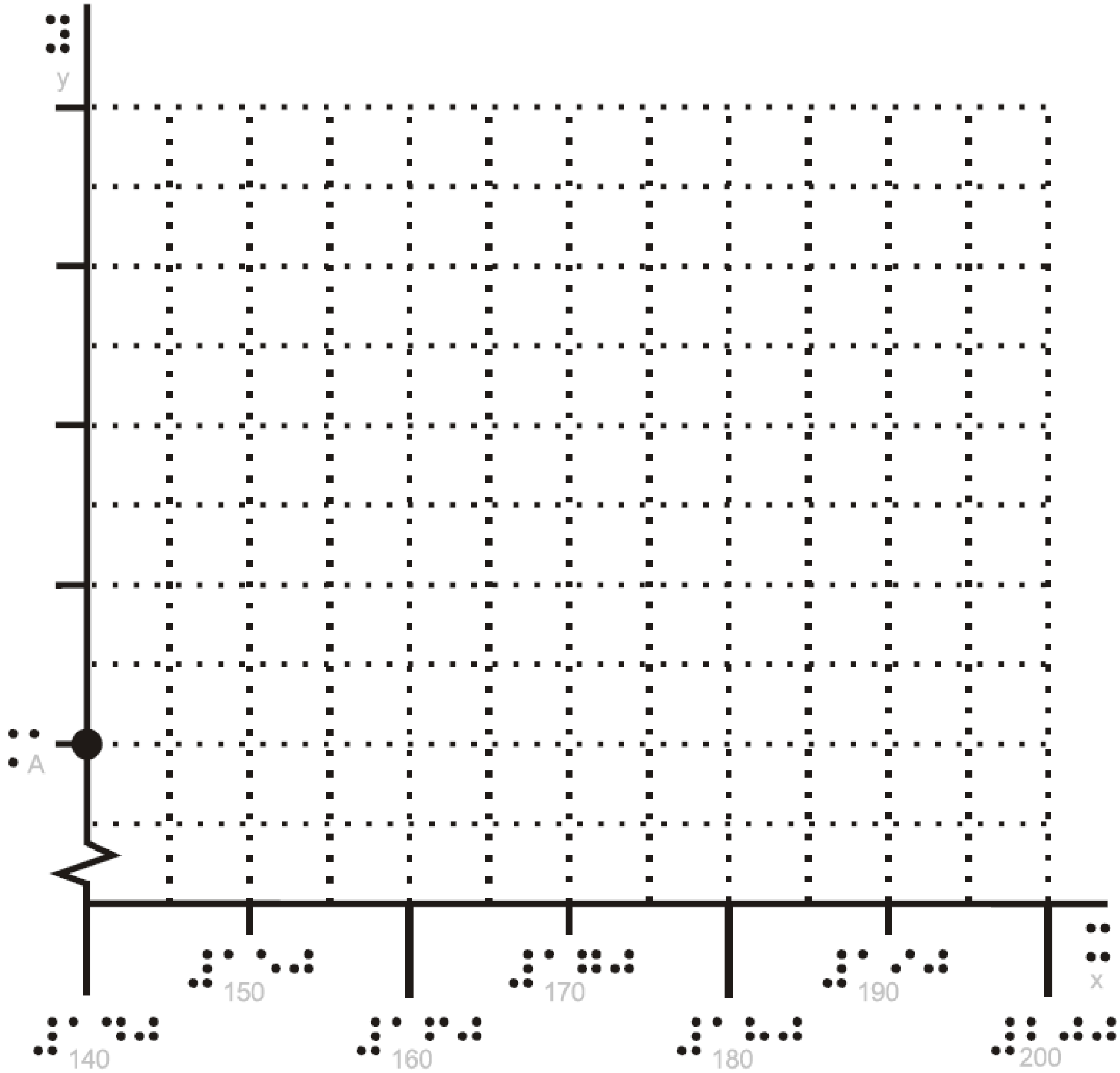
Question 4: 2 points

In the formula for calculating the final height, fill in 180 cm for the height of the father. Then you can also write the formula as:

final height = 0,5 * height mother + ...

Complete the above formula by filling in the correct number on the dots

Drawing 1: For this assignment the drawing below needs to be made tactile for the student to be able to feel it. (in raised lines)


y
A
x
140
150
160
170
180
190
200

4 Teaching Mathematics to Blind and Vision-Impaired Students: The Polish Context

4.1 Introduction

In this section of the Report, we present the organisation of formal education in Poland, including the education of students with visual disabilities. We analyse statistical data on the education of students with visual disabilities from the two school years - 2016/2017 and 2012/2013. We analyse them comparatively and present the observed changes. We present research conducted in Poland on the use of ICT in Polish schools, and research aimed at the use of ICT in the mathematics education of blind and low vision students, conducted by the Institute of Mathematical Machines (IMM) in 2014-2017, and continued in 2018 by NASK PIB[1]. Based on the results of the research, we describe the state of computerization of inclusive maths education as well as the needs, expectations and preferences of teachers and students with visual disabilities regarding ICT support. We present the ICT most commonly used in inclusive mathematics education in Poland. We analyse the data of the Central Examination Board (CEB) from the final high school exams in terms of the number of students with visual disabilities who took the final exams. We also analyse final exam questions in terms of their accessibility, and (where necessary) show how any deficiencies may be rectified to ensure that the question may be answered by those in this demographic.

4.2 The system of formal education in Poland

The school system in Poland is currently undergoing reform. The new organization of education has been introduced in September 2017 and will be gradually implemented until 2020, as shown in Figure 9. At present, there is an eight-year primary school, and after the primary school, young people can choose a four-year general high school, a five-year technical school or a two-stage industrial school. There are also post-secondary schools, which can be attended after graduating from high school or technical school, without the obligation of having a high school certificate.

[1] On February 2, 2018, the Institute of Mathematical Machines (IMM) in Warsaw has been included in the NASK Research and Academic Computer Network. The EuroMath (Erasmus +) project launched by IMM is continued at NASK PIB.

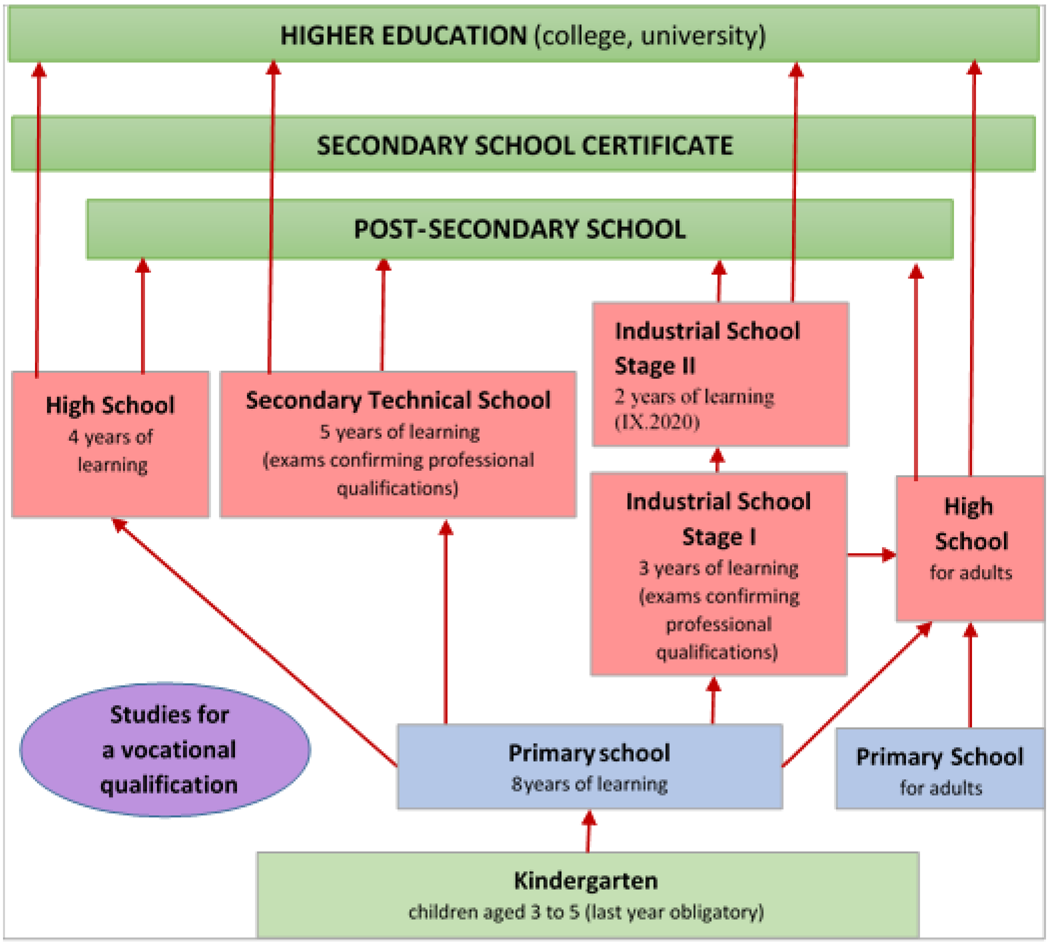


Figure 9 Diagram of education structure in Poland after the commencement of the education reform in 2017.

Source: own work based on The Act of 14 December 2016 - Educational Law (Journal of Laws of 2017, item 59)

Primary school is attended by children from the age of 6 or 7 depending on the parents' decision. The previous system of education, presented in Figure 10, was based on a six-year elementary school, a three-year junior high school, a three-year high school, including profiled secondary school, four-year technical school and a three-year vocational school. There were also post-secondary schools.

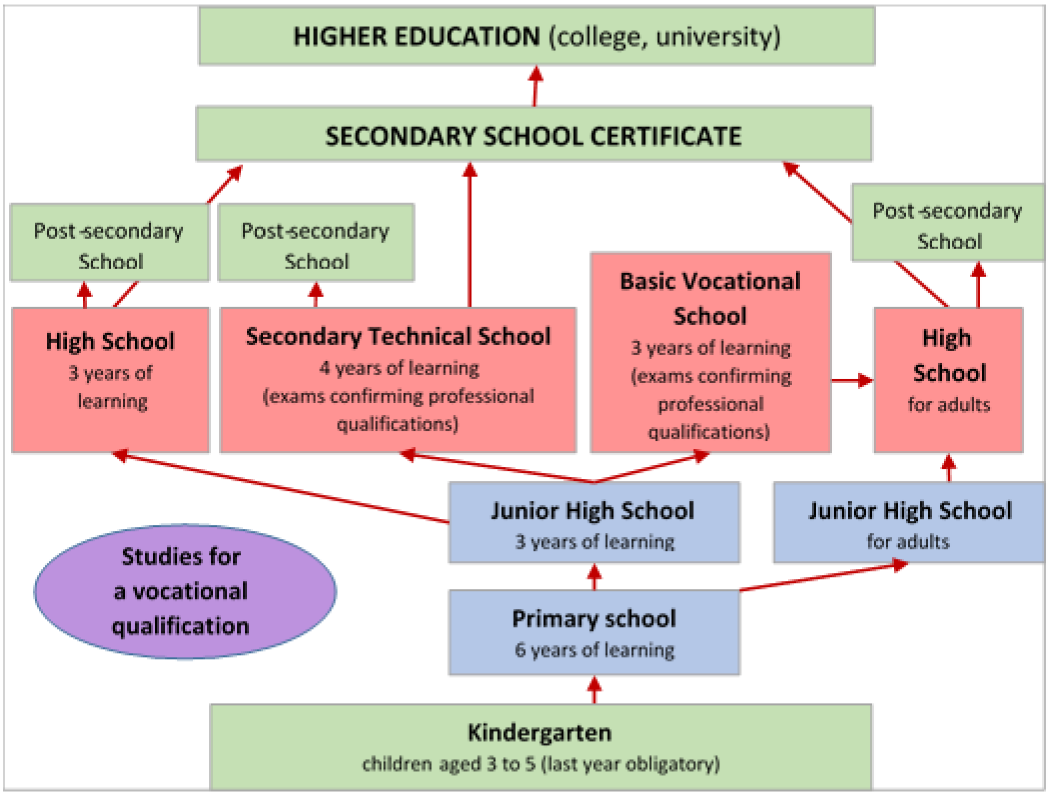


Figure 10 Diagram of the education system in Poland before the reform in 2017

Source: own work based on The Act of 14 December 2016 - Educational Law (Journal of Laws of 2017, item 59)

In the education reform, junior high schools and vocational schools were abolished, and were replaced by eight-year primary schools and two-stage industrial schools which are to cooperate with enterprises in the process of youth education. Technical schools and high schools have been extended by one year (respectively to 5 and 4 years). Since 2016 parents may decide whether their children start primary at the age of 6 or 7.

The next chapter presents a statistical picture of Polish school education regarding children and youth with visual disabilities.

4.3 Blind and low vision students in the Polish education system

4.3.1 Nationwide statistical data

In Poland, the idea of inclusive education is being implemented at all levels. Inclusive education, according to the postulate of the UN Convention on the Rights of Persons with Disabilities, means a situation in which a mainstream school is prepared to receive and meet the educational needs of each student. The organization of such an education system does not assume the need to create a special infrastructure, but to adapt the existing one to the requirements of students with various needs. At present, in connection with the ratification of the aforementioned convention by Poland in 2012, the assumptions of the idea of inclusive teaching began to determine the shape of educational reforms. Much is changing for the better, although it is still difficult to talk about the full inclusiveness of the Polish school system.

Since 2010 students with special educational needs, including those with disabilities, can fulfil their school obligation in any type of institution. In order to provide formal support for a student, a decision has to be made by a psychological and pedagogical counselling centre on the need for special education. After obtaining it, the student has the status of a person with disability, which entitles him/her to receive help appropriate to his/her needs. In practice, this provision applies to decisions regarding three types of disability - autism, Asperger's syndrome and multiple disabilities. In connection with this practice, students with visual disability attend special or integration education institutions. Students with low vision with a lesser degree of visual disability attend integration schools or mainstream schools with integration classes. Integration upbringing and education is an educational offer for students with various types and degrees of disability. The Polish model of integrated education was created and implemented before the Convention was signed in 2012, and it is still in force, along with the gradually expanding system of inclusive education. The model was based on the Act on the education system of September 7, 1991 (Journal of Laws No. 67 of 1996, item 329, as amended), which ensured the possibility of this form of education in all types of schools, in accordance with individual development and educational needs of this group of children. According to Polish law, integration classes are created when three, four or five students with disabilities can create a 'core' of the integrated class. In the integration class there is a special educator who supports the process of education of students with special needs. Changing the education system towards a more inclusive one is not just about implementing proven foreign or domestic solutions. It requires, above all, cultural change, transformation in the way of thinking about special educational needs, including allocations of appropriate financial resources.

In summary we can conclude that in Poland students with visual disabilities can study and learn in mainstream schools, in mainstream schools with integration classes, in integration schools and in special educational centres for blind and low vision children.

In its report [39] for the year 2017, the Central Statistical Office (CSO) states that in the 2016/2017 school year, the number of all pupils with visual disabilities, blind and affected with low vision, amounted to 7,506, including 251 blind students and 7,255 low vision students (Table 10). In mainstream schools, including integrated schools or those with integrated classes, there were 75 blind students and 6,539 students with low vision. Special centres were attended by 176 blind students and 716 students with low vision. The percentage of blind students is only 3% (Table 11) of pupils with visual disabilities who use mainstream education. The majority, 97%, are students with low vision, using public education (including integration schools and classes). In special schools, the proportions between blind and low vision students are less extreme and amount to 20% and 80% respectively, which confirms the fact of unpreparedness of Polish public schools for educating blind students, with concurrently well-organized and functioning Special Centres for blind and low vision children. In Poland there are 11 state-run Special School and Educational Centres for Blind and Visually Impaired Children (SOSWs - Specjalne Ośrodki Szkolno-Wychowawcze dla Dzieci Niewidomych i Słabowidzących), among them one centre (in Warsaw) is intended only for children with low vision, while the Centre in Radom does not teach mathematics, having blind or low vision students with multiple disabilities, including intellectual disabilities. SOSWs have dormitories in which students live, having the possibility to go home on weekends as well as for vacations.

SOSWs (apart from the Centre in Radom) provide education for children and youth at all levels, in some SOSWs there are 2-year post-secondary schools, gradually being closed down as a result of the reform of education. In the school year 2016/2017, SOSWs were attended by 187 blind students and 372 students with low vision. The greater number of blind students in SOSWs in Table 12 in relation to the number of blind students in special schools in Table 10 results from not including the post-secondary schools in Table 10 (data for schools at this level are not separated in the cited CSO reports). It is worth noting in Table 10, the decreasing number of students with visual disabilities, both blind and those with low vision, in schools with higher than the basic level of education, both in mainstream and special schools.

		School year 2016/2017		School year 2012/2013	
	School type	**blind**	**low vision**	**blind**	**Low vision**
1	Basic vocational school	1	121		
2	Primary schools	31	3,697	42	2,455
3	Special primary schools	62	207	79	188
4	Junior high schools	19	1,694	27	1,450
5	Special junior high schools	37	165	52	211
6	Basic vocational schools	1	121	1	93
7	Special basic vocational schools	16	75	24	124
8	High schools	18	528	20	402
9	Special high schools	29	114	31	169
10	Technical schools	6	499	Lack of data	235
11	Special technical schools	32	155	47	185
	Sum total in all schools	**251**	**7,255**	**323**	**5,512**
	Sum in public schools	75	6,539	90	4,635
	Sum in special schools	176	716	233	877
	Total number of students with visual impairments in all schools		**7,506**		**5,835**

Table 10. The number of blind and low vision students in public and special schools in Poland

Source: own work based on CSO Education and Upbringing reports [39] [40]

	School year 2016/2017		School year 2012/2013	
	% of students		**% of students**	
	blind students versus all students with visual impairment	low vision students versus all students with visual impairment	blind students versus all students with visual impairment	low vision students versus all students with visual impairment
In public schools	3%	97%	2%	98%
In special schools	20%	80%	21%	79%

Table 11 Participation of Polish public and special education schools in the education of students with visual disabilities in the 2012/2013 and 2016/2017 school years

Source: own work

	School year 2016/2017		School year 2012/2013	
School type	**blind**	**low vision**	**blind**	**low vision**
SOSWs (including post-secondary schools)	187	372	209	517

Table 12 The number of blind and low vision students in Poland's SOSWs in the 2012/2013 and 2016/2017 school years

Source: own work based on CSO Education and Upbringing reports [39] [40]*)*

In addition to the data for 2016/2017 school year, the tables 10, 11 and 12, include for comparison the corresponding data from the CSO report for the 2012/2013 school year. It can be seen from the data of Table 10 that in the period of 4 years, from 2012 to 2016, the number of students with visual disabilities increased from 5,835 to 7,506. This resulted from an increase in the number of low vision students, while the number of blind students had actually decreased from 323 in 2012 to 251 in 2016. These quantitative phenomena occurred at the time when the population of Poland decreases from 38.06 million in 2012 to 37.95 million in 2016. The analysis of the reasons for the growth in the number of visually impaired students in Poland in the last 4 years, at the time when the total population of Poles decreased, is beyond the subject matter of this publication. A positive although not intensive trend that can be seen from the data in Table 10, is the reduction in the number of blind and low vision students attending special schools. This trend is more visible in the case of low vision students - from 16% of all low vision students who attended special schools down to 11%, hence a reduction of 5%. In the case of blind students, the corresponding reduction is 2%, from 72% to 70%. It can be assumed that the reason for this positive occurrence of a 'march' towards inclusive education in Poland is the constantly improving readiness of public schools, their teaching staff and technical equipment, including ICT, for the education of students with visual disabilities. Table 12 provides further proof of the development of inclusive education in Poland. Over the period of 4 years under discussion, the number of students with visual disabilities attending special schools had decreased by 20%, while those attending SOSWs decreased by as much as 23%. Among organisations studying these trends is the TRAKT Foundation of Polish Blind and Low Vision People (www.trakt.org.pl), which in 2013-2014 conducted qualitative research primarily among parents of children with visual disabilities regarding their opinion on sending a child to a special school. Parents of blind or low vision children without multiple disabilities prefer to send a child to a public school, so that they do not lose contact with the child and that the child does not lose contact with siblings. The role of siblings attending the same school is also helpful. Not only better preparation of mainstream schools for inclusive education contributes to this social trend. The students' competences concerning ICT have also increased, and there are also many ICT supportive tools that schools and students benefit from, as we present further on in the study. Blind students are getting better at learning and thanks to the support of ICT, they are becoming more and more self-reliant.

4.3.2 The needs for and use of assistive ICT tools

In Poland, the subject of mathematics teaching is the least supported by ICT in relation to other subjects, both in mainstream schools and in special schools. The first research was started by the IMM in 2014 [41]. It concerned the needs in the area of ICT supporting mathematics lessons. These were preliminary studies, providing orientation in the intensity and directions of needs, necessary to develop a platform model with ICT tools supporting both mathematics teachers and students with visual disabilities. The platform named PlatMat, was created in 2015. Further research conducted by IMM in the laboratory contact of teachers and students with the PlatMat model (testing, initial training) in 2015 concerned the teachers' assessment of the usefulness of the applications and their functions available in PlatMat [42] Subsequent in-depth IMM research, using measurable criteria, related to the measurable effects of the use of supporting tools in the work of the teacher and students, was carried out in 2017 during the pilot implementation of the PlatMat tools in practical educational activities in 3 school facilities. As several years have passed since the first IMM research in 2014, research was carried out at the end of 2017 and in early 2018, launched by IMM, continued by NASK PIB, were aimed at examining the current situation in terms of ICT used by teachers and students and current ICT needs.

Published research on the digitization and application of ICT in mainstream and special schools

We could not find any published information on research concerning ICT supports for maths learning at school and at home by students with visual disabilities in Poland, other than that referring to research conducted since 2014 by IMM, and then by NASK PIB. However, there are publications on research regarding ICT supports for students with visual disabilities or students with special needs, in general, in school education. We will be following on with a discussion on these research sources. Most of that research is based on teachers' statements, and to a large extent is focused on the ICT supports for teachers themselves. Karol Bidziński [43] examined the frequency of ICT application in didactic activities in relation to students with special needs lists five didactic activities: searching for methodical information; preparing teaching aids; preparing multimedia presentation;, disseminating students' achievements on the WWW network; and organizing work for students on the e-learning platform. It is worth noting that only one of these activities, organizing work for students on the e-learning platform, concerns the ICT supports for students. Out of the 38 collected statements, 25 describe the frequency of use as very low.

Type of pedagogical activities	Apply	Seldom	Sometimes	Often	Regularly
Organising work for students on e-learning platform	38	25	6	5	2

Table 13 Frequency of ICT use by teachers in pedagogical activities

Source: own work based on [43]

There is no justification given in the publication, as to why this type of ICT support for students was examined and whether the need to support students through Moodle or another system of LMS type is the most important in the educational process aimed at students with special needs. It can be assumed, however, that it was about supplementing students' learning through remote study, self-reliant or supported by consultations. The research indicate that students seldom use e-learning tools. The author does not explain whether it is about their use during lessons or for independent work outside the school. The results of the research presented in this publication show a positive image of the teaching process being improved through the use of ICT, but it should be added that this conclusion applies only to the part of this process regarding the preparation of teaching materials and lessons, and not the ICT support of students with special needs in the classroom. A similar picture of the good teachers' mastery of ICT tools needed to create materials for lessons (e.g. student assignment cards, tests) results from unpublished research carried out by IMM/NASK PIB in EuroMath Project, described further on.

Studies on supports for blind and low vision students, conducted at the Maria Grzegorzewska University (Akademia Pedagogiki Specjalnej im. Marii Grzegorzewskiej) in the years 2016-2017, were presented in the work Paplińska [44]. Paplińska's research was conducted in 35 special schools and 36 mainstream schools.They concerned the general education of these students, without distinguishing the teaching subjects. In addition to teachers, 24 students with visual disabilities took part in the study, in equal numbers from mainstream schools and special schools. It is worth paying attention to students' statements because they sometimes see their problems in using ICT more clearly than their teachers. Only half of the students declared the need to use school-owned specialist equipment. The conclusions from the Śmiechowska-Petrovskij research [45] confirm the fact that there is insufficient amount of computer and specialist equipment as well as specialist software that can support visually impaired students in Polish schools. In the majority of cases studied, the materials for the lessons continue to be prepared in a traditional form - printed materials, including Braille. Only a quarter of students receive materials for lessons in a digital form, and concerning mathematics - only 2 students declared that they received materials in this form. The use of ICT in classroom for writing tests was declared by 4 students from 24; 3 students declared that they were solving tests in a digital form; and 6 students indicated that they used a computer for writing during lessons. The research did not distinguish between either the statements of blind and low vision students, nor the subjects being taught. The scope of the research was the use of computer and specialist equipment, and concerning software - only that using sound and screen magnification programs. The publication does not refer to the commercially available specialist programs that support students with visual disabilities and to their use in schools. If they were used, they would be covered by the research. The gap between technological possibilities and educational practice results from the lack of continuing teacher training in dynamically developing ICTs and the lack of systemic solutions regarding the selection and implementation of ICT supports in the Polish educational system.

It is worth referring to the information contained in a report published by the Supreme Audit Office (NIK) in 2017 [46]. It concerned the digitization of Polish schools within the 'Digital School' program, which covered 425 schools out of 30,000 existing in Poland. According to the report, in the second semester of 2015/2016 school year, in the schools under control, 50.3% of lessons on non-IT subjects were conducted using ICT, out of which only 17.8% of lessons were conducted using ICT in direct student work, in 32.5% of these lessons teachers used ICT to support didactic activities.

This overview of the published information on the use of ICT supporting both students with normal vision and those with visual disabilities can be summarized with a conclusion that the level of computerization of lessons (not teachers) in Polish schools is low. There are several reasons resulting in this situation. The report indicated that teachers as the main reason give insufficient stock of ICT devices in schools. From the Supreme Audit Office findings, as well as IMM information obtained in interviews with teachers, a significant problem is the lack of technical support for teachers. The subject teacher does not have the same IT competencies as the professional ones, and it is difficult to assume that teachers will acquire them even after several training sessions. The teacher is concerned about the use of ICT in lessons without IT support. According to the Supreme Audit Office report, such support is available only in 1/5 of schools (see Figure 11).

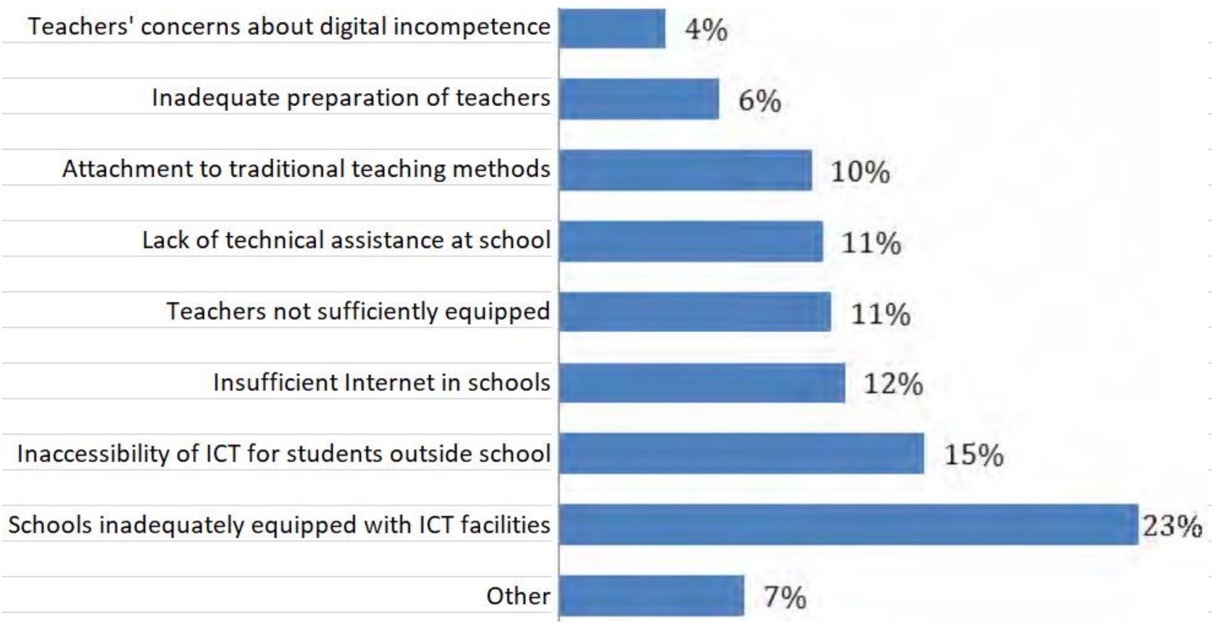


Figure 11 Main barriers to the use of modern ICT in schools

Source: own work based on [46]

Research on the ICT needs and teacher's opinions on the ICT impact on math education

Research on maths education, focusing on the needs for assistive ICT applications, and ICT applications used in practice in inclusive maths education, started in the Institute of Mathematical Machines in 2014, and after the inclusion of IMM in NASK PIB in 2018, research on this subject has been continued by NASK PIB as part of the EuroMath project (Erasmus+).

4.3.2.1.1 Research on the needs for ICT supports

The research was carried out in 2014. It was conducted in the form of surveys using electronic questionnaires developed in Google Forms and using the method of individual telephone interviews, and it included teachers of mathematics, blind and visually impaired students and their parents, from mainstream schools, schools with integrated classes and specialist upbringing-educational centres. The research included respondents from all over Poland. A total of 142 respondents were surveyed, of which 104 completed an electronic questionnaire and 38 were interviewed on the phone.

Research and conclusions from the research have been presented in Rubin, Faderewski, Mikułowski [41].

The surveys have shown, among other factors, a high level of computerization, familiarization with IT devices and Internet use among teachers, as well as ignorance of Braille by a large number of teachers:

- all teachers use a computer, and as many as 71% use one every day;
- almost all teachers use the Internet, and 74% do it every day;
- 40% of teachers browse the Internet in search for materials for teaching maths to people with visual disabilities;
- a very large group of teachers in the survey do not know Braille (48%); teachers who do not know Braille are from mainstream schools with or without integration classes.

Teachers are ready to use new technologies helping them in teaching and interacting with blind and low vision students:

- teachers decisively (91%) want to learn new technologies useful in teaching mathematics;
- the great majority (90%) would like to implement and use IT tools in their work if they were properly adapted to their needs.

A somewhat lower level of use of computer and IT devices, as well as use of the Internet is demonstrated by the surveyed students (a larger percentage among the low vision students, smaller among the blind) and the share of students who do not know Braille is similar to that of the teachers:

- all students use computers, and 64% of them do so on a daily basis (this includes the decisive majority of upper secondary and junior high school students, mostly the low vision students);
- also all students use the Internet, and 64% of them use it every day;

- only 14% of students browse the Internet in search for materials and aids to learn mathematics;
- the majority of students (51%) use a tablet or smartphone device on a daily basis - however, they are mainly low vision students (almost 87%);
- 47% of students do not know Braille at all, the remaining 53% of students know Braille at different levels.

This brief outline shows that the situation concerning computerization and IT use among the teachers and students and their readiness to face and accept new technological challenges adapted to teaching and learning mathematics, with the concurrent considerable unfamiliarity of Braille recorded in both groups - teachers and students, create a solid basis for the adoption of new IT solutions.

The expectations of teachers and students identified in the survey are:

- possibility of collecting digital versions of convex drawings by teachers so that they can be easily printed (e.g. on a 3D printer);
- creating a database of models of solid figures for printing on a 3D printer;
- editors for a student should be able to express text in bold, change the font size, and change the colour of background and text;
- implementing a digital model of the coordinate system;
- digital numerical axis on which intervals and limiting points can be marked;
- automatic archiving of the material saved during the lesson, so that the student (or teacher) can return to it, for example when students make a mistake and delete data;
- introduction of the possibility to set the colours of the mathematical notation syntax;
- introduction of profiles that will enable to set colours for individual elements of the notation (e.g. numbers, symbols, actions).

In addition to open-ended questions, questionnaires contained selection options concerning new ICT solutions such as: virtual cubarithm which allows blind students to learn arithmetic operations in a written manner; formula editor for blind students; navigator through formulas; function graph generator; acoustic viewing of mathematical graphics.

Figure 12 shows the respondents' level of acceptance of the proposed new ICT solutions which constituted selection options in the questions.

The results of this research indicate that there is a compelling need for new ICT solutions focusing on students with visual disabilities, especially blind students, constituted the justification for IMM's launching of research and development work on PlatMat supporting technology, conducted in close cooperation with teachers and students.

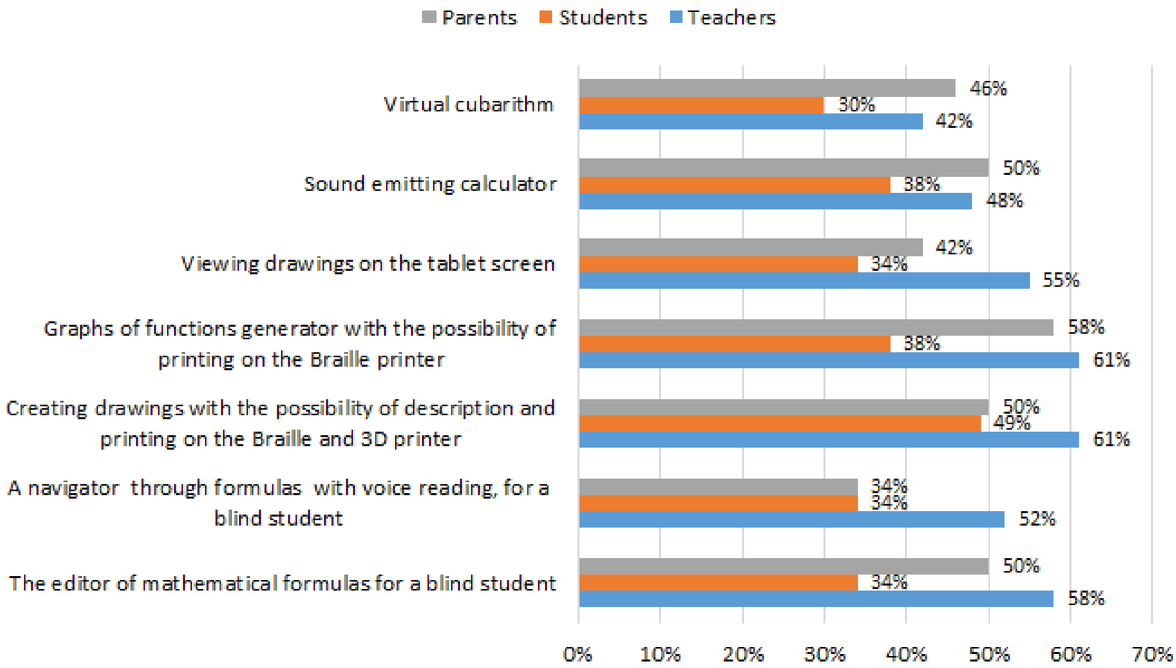


Figure 12 Level of acceptance by teachers, students and parents of the proposed new ICT solutions supporting inclusive mathematics education

Source: own work

4.3.2.1.2 Research on the assessment of usefulness of ICT solutions supporting inclusive mathematics education

In 2015, new ICT solutions developed by IMM to support inclusive maths education were made available for testing and assessment of their expected usefulness. 24 mathematics teachers from 8 educational institutions in Poland, teaching students with visual disabilities, took part in the study. It comprised quantitative and qualitative surveys, which were presented along with the results in Rubin [42]. Quantitative research concerned the completeness of the tested solutions. The results of quantitative research on the most important ICT solutions aimed at supporting teachers and students are presented in Table 14. It is worth emphasizing that the opinions of teachers were formulated on the basis of testing, not actual use, because it was a stage of testing model solutions, and not their implementation. The new ICT solutions that were subject to assessment were:

- Creating and printing mathematical aids (black print, Braille, typhlographics, 3D);
- Universal (for blind and low vision students) multimedia, interactive mathematical documents in the EPUB3 format, containing formulas in MathML notation, math vector graphic in SVG format, MP3 recordings of teacher or student comments, active links to video materials e.g. on YouTube;
- Virtual Cubarithm (Windows application) for learning arithmetic calculations in writing by both blind and low vision students, including the possibility to customize the user interface to user's needs;

- Acoustic reading of graphics for their preliminary recognition by a blind student, which consists of semantic synthetic audio reading of graphics and its elements recorded by the teacher or as metadata generated automatically during the recording of graphics based on their parameters (e.g. vertex coordinates) and providing sound to function graphs as well as technical sound, the so-called ‘audio-icons’, e.g. informing about the touch gesture moving outside the area of the figure;
- Multimedia teacher's (explanatory) comments and student’s comments (with questions or explanations);
- Navigation through formulas, from keyboard or by gestures, assisted by a synthetic audio reading of formula elements;
- Semantic reading of formulas with synthetic voice;
- Available EPUB3 mathematical document;
- Available EPUB3 multimedia maths resources on the portal;
- Document areas to be edited by the student, and other content only for reading;
- Remote desktop - monitoring the screen content of the selected student displayed on the teacher's screen.

Do you agree that the following tools accelerate and facilitate the teacher's preparation of classes and work with visually impaired students in the process of mathematics education?	**Yes**	**No**	**I have no opinion on the matter**
Printed math aids (in black print, Braille, 3D)	19	0	5
EPUB3 universal mathematical documents	20	2	2
Cubarithm for learning written arithmetic calculations	17	0	7
Do you agree that the following tools increase self-reliance and pace of work of the visually impaired students in acquiring mathematical competences?	**Yes**	**No**	**I have no opinion on the matter**
Acoustic readout of graphics	15	0	9
Multimedia comments	16	0	8
Navigation through formulas supported by sound reading	15	0	9
Semantic reading of formulas	15	0	9
Availability of EPUB3 mathematical document	16	0	8

Do you agree that the following tools have a positive impact on the effectiveness of cooperation between a teacher and a visually impaired student in the process of teaching/learning mathematics?	**Yes**	**No**	**I have no opinion on the matter**
Available, multimedia, online EPUB3 mathematical resources	20	0	4
Areas of the document dedicated to student activities	17	0	7
Remote desktop	18	0	6

Table 14 Teachers' assessment of usefulness of new ICT solutions supporting inclusive mathematics education

Source: own work

The conclusions from these studies indicate that the new solutions are highly supported and that their role in accelerating the learning of mathematics and increasing the student's self-reliance is appreciated. Due to the fact that the model solutions were tested by teachers in a laboratory conditions and only after training in the use of PlatMat, and not in the conditions of actual teacher's work, with experience of the PlatMat application, there can be observed many opinions expressing uncertainty about the usefulness of some new solutions, e.g. semantic reading of formulas or navigation through the formula combined with the reading. Subsequent research, this time of the PlatMat prototype in the conditions of the pilot implementation, described in the next chapter, will show the teachers' positive attitude on this matter.

4.3.2.1.3 Research exploring the measurable benefits of ICT supports in teacher's and student's work

The purpose of the research was to determine the measurable benefits of using PlatMat tools for a mathematics teacher, a blind student and a low vision student. The research was carried out in 2017 following a several-month pilot implementation of the PlatMat tools in three educational institutions - SOSW in Krakow, SOSW in Warsaw and Junior High School No. 3 with Integration Classes in Siedlce. Because not all students in the classrooms were equipped with laptops, the pilot courses were conducted primarily as levelling courses, individually with single students or in small 2-3 person groups of students. The levelling classes were also conducted via the Internet, as well as on-going online consultations regarding homework. Five mathematics teachers took part in the qualitative study, including three specialising in typhlo-pedagogy, as well as eleven students - four blind and seven with low vision. The research was conducted in the form of a survey and individual interviews. The survey was composed of three parts - on the teacher's work, blind student's work and work of low vision student, and the fourth part with the criteria and measures of the assessment. The research was described in the internal IMM report [47].

We asked teachers about measurable benefits in the following processes:

Process 1

1. Preparation of work cards and aids in the form of printouts for solving the problems by blind and low vision students, taking into account the time of preparation by methods used so, considering every student (variety of disabilities) in the classroom.

Process 2

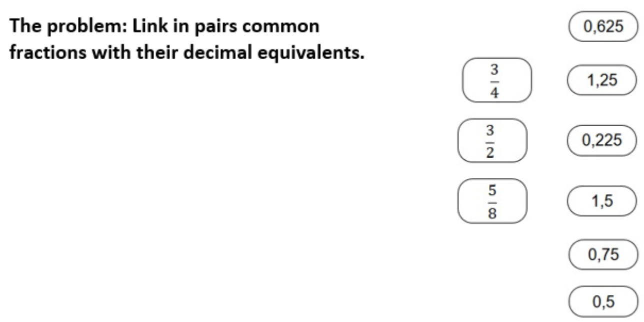
1. Distribution of cards and aids versus the time of dictating and distributing by the foregoing methods.
2. Taking/receiving solved problems/cards.
3. Checking by the teacher, indicating errors (in PlatMat – with special comments) and passing to the student for correction
4. Checking after correction.

Process 3

1. Teacher's aids for helping students (monitoring of student’s desktop in the classroom, online help at home/boarding house):
    a. in correcting student’s work;
    b. in supplementing student’s learning (weaker student, student after a longer absence from school due to illness);
    c. in the initial solution of the problems.

We asked the low vision students about measurable benefits in the following operations:

1. Calculation of different received (arithmetic, power, polynomials, roots) one-dimensional and two-dimensional one-liner formulas in the student’s preferred editor and by the method used so far;
2. Writing/solving a set of equations in the editor preferred by the student and by the method used so far;
3. Solving/transforming multi-line formulas in the editor preferred by the student and by the method used so far;
4. Solving a task with content with parameters for calculations in the editor preferred by the student and the method used so far;
5. Solving the ‘pairing’ type testing problems;
   In the Figure 13 there is an example of a ’pairing’ type testing problem, displayed for



   low vision students on an interactive board, and for blind students sent to their computers as an EPUB3 document and served using the QWERTY keyboard

Figure 13 An example test task ‘pairing’

Source: own work

6. A geometrical problem to calculate ‘something’ in a figure:
    a. with a figure in the document;
    b. with a figure to be drawn (function graph, figure drawing).

We asked the blind students as well as low vision students, with small additions to questions, about measurable benefits concerning the following operations:

1. Calculation of various (arithmetic, power, polynomials, roots) received one-dimensional and two-dimensional one-liner formulas in the editor preferred by the student, and by student's preferred method used so far;
2. Writing/solving a set of equations in the editor preferred by the student and by student's preferred method used so far;
3. Solving/transforming multi-line formulas in the editor preferred by the student and by student's preferred method used so far;
4. Solving a problem in text form with parameters for calculations in the editor preferred by the student and by student's preferred method used so far;
5. Solving the ‘pairing’ problem (in the version for a blind student with the numbering of areas to be paired);
6. Geometric problems for calculating ‘something’ in the figure, with auxiliary material in the form of typhlographic print on swell paper;
7. Correction of problem solution (modification of simple and multiline formulas with finding the place of error indicated by the teacher by means of a special commentary:
    a. by using the navigator, selecting the erroneous part of the formula and editing it in the editor preferred by the student;
    b. by placing the entire solution directly in the preferred editor and navigating through the formula in the editor;
8. Correction of problem solution (modification of simple and multiline formulas with finding the place of error indicated by the teacher by means of a special commentary in the formula editor chosen by the student).

For each question the respondents had the suggested assessment criteria and specific measures.

Assessment and measurement criteria

What we measured and the method we used:

1. Time (shorter, longer, the same as so far);
2. Number of printouts (greater, smaller, unchanged);
3. The number of errors (greater, smaller, the same as before) and possibly the causes of errors;
4. The possibility of execution (whether it had been at all possible to perform a given operation or provide appropriate help) - Yes/No;
5. Completeness of the solutions to problems received - the number of problems fully resolved (including those with errors) in relation to the number of all problems;

6. Ease of work:
    a. of a given student (greater, smaller, the same as so far);
    b. of students in a classroom (taking into account the noise from Braille printers): (greater, smaller, the same as so far);
    c. of a teacher: (greater, smaller, the same as so far).

    When formulating the assessment of benefits, we had to take into account:

    - choice of teacher-friendly options in the teacher's tool; in the tool for low vision students and the one for blind students – choice of options convenient for each student;
    - possibility of work without leaving the PlatMat environment for other tools;
    - printing of aids on swell paper at home, and heating it at school on the heater;
    - possibility of monitoring student's work from the teacher's desktop in the classroom or at home;
    - configurability of PlatMat tools to the student's needs (profiling);
    - remote teacher-student cooperation via the internet.
7. Communicativeness:
    a. the effectiveness of handling student's work by the teacher (quick reaction to errors and the possibility of accurately indicating and describing the error) – (greater, smaller, the same as so far);
    b. availability of teacher's help from the student's point of view (student can call for help as soon as it’s needed or arrange for help) – (greater, smaller, the same as so far);
    c. sending math documents via Wi-Fi and by the Internet, to a cloud, e.g. to Google Drive.
8. Student's self-reliance in areas where it was not possible or was possible to a lesser extend with existing methods (greater, smaller, the same as so far).

    We had to take into account:

    - Removed need for the student to dictate solutions to the teacher/parent in a situation of very poor vision or not knowing Braille (in a mainstream school - inclusiveness of education);
    - The possibility of self-reliant arithmetic operations in written form by a low vision student and a blind student;
    - Self-reliant recognition understanding of the structure of a complex formula (thanks to navigation through the formula);
    - Self-reliance in the modification of the formula by a method preferred by the student (the whole formula can be loaded into the editor and using cursor go to the place of error or select for editing the fragment found using the navigator);
    - Chat in Braille.
9. Work in the classroom to meet various needs of students with different levels of visual disability (mixed group of students in inclusive education – students with normal vision, blind students and low vision students with different degrees of visual disability)
    a. time spent on preparation of materials for the lesson (less, more, the same);
    b. time spent on checking and indicating/notifying about errors (less, more, the same);

c. monitoring of each student during their work (terms to choose from: more precise, faster, complete, possible, convenient, other term).

The main benefits identified in the study and described in detail in the Report from the survey [47] are as follows:

- improvement of younger students' efficiency concerning use of IT, which shortens the time of performing math operations (mastery of keyboard shortcuts);
- shortened time of math operations performed by low vision students;
- greater comfort of low vision students as a result of tools profiling possibility;
- increased efficiency of student's communication with teacher as a result of legibility of student's notes;
- increased students' self-reliance, among others as a result of semantic reading of formulas, recognizing the structure of the formula, and own notes transparent to oneself;
- the number of errors made by the students was not greater, while it was clearly lower in the case of problem solving by 'pairing', and where the problems were in the form of text with parameters for calculations;
- faster correction of calculations by low vision students, among others as a result of comments posting by teacher, indicating/describing the error;
- better communicativeness, student's ease of work and aid effectiveness as a result of student's screen being monitored by the teacher;
- shortened time of preparing work sheets, significantly shorter in cases of problems with fractions, roots, sets of equations and special characters, the use of which is quite troublesome in the Word editor;
- increased teacher's ease of work as a result of collecting of all the necessary functions in one tool instead of having to use many different tools;
- increased possibility of helping student, and also in a more accurate way.

Blind students expressed the opinion that use of supporting ICT tools in teaching maths should start at an early stage of education, before the use of traditional methods based on Braille and touch (two hands) becomes a habit, which hinders, for example, full acceptance of new acoustic methods of learning about and recognising mathematical graphics.

Teachers are of the opinion that the use of uniform ICT tools, including PlatMat, by all students in the integrated class at maths lessons, can have a positive effect on the ease of functioning of students with visual disability, as they diminish the differences in access to mathematical content.

The postulate of mathematics teachers to include supportive teachers in future cooperation in the implementation of ICT in school practice was also noted.

Research on ICT applied in inclusive mathematics education

At the turn of 2017/2018, for several months, within the framework of EuroMath project[2], there were conducted qualitative surveys on the use of ICT in the mathematical education of students with visual impairment. The aim of the study was to acquire knowledge about the

[2] The EuroMath Erasmus+ project under the heading 'EuroMath - increasing the support of teachers and visually impaired students with innovative ICT in inclusive mathematics education', is being implemented by the project coordinator, NASK PIB, in the period 2017-2020.

extent to which ICT supports meet the needs of teachers and visually impaired students in teaching/learning mathematics in Polish schools.

The invitation to the survey, together with the questionnaires, was sent by post and e-mail to all SOSWs and several schools with integrated classes, based on the attendance list at the PlatMat technology conferences. The EuroMath project implementers personally delivered surveys to the public schools. The criterion for choosing the school was its close location in relation to the implementer's place of residence or cooperation with the school in the past. 29 mathematics teachers from 15 schools from all over Poland took part in the survey. Among these 15 schools there were 6 mainstream schools without integrated classes, 3 schools with integrated classes and 6 Special School and Educational Centres (SOSW). The distribution of respondents in particular types of schools is presented in Table 15.

Item	Type of school	Number of schools	Number of respondents
1.	Mainstream	6	8
2.	mainstream with integrated classes	3	7
3.	SOSWs	6	14
	Sum:	15	29

Table 15 Distribution of the number of respondents in particular types of schools

Source: own work

Two questionnaires were prepared for the survey - one of them concerned ICT supports applied in mathematics lessons, and the second - the use of interactive boards and monitors during math lessons. The questionnaire analysing the use of ICT supports included six questions regarding:

- school’s network infrastructure and Internet use;
- school's ICT devices;
- computer equipment used by mathematics teachers;
- computer equipment used by blind and low vision students
- ICT used by teachers in math lessons.

The questions, the selected answers and open responses, as well as additional open questions asking for detailed information that we were most interested in, were in questionnaires. Table 16 shows the collected responses from completed ICT questionnaires. Table 17 shows the collected responses from those completed on whiteboards and monitors questionnaires.

Questions and answer options	Number of answers	Collected answers to open questions: what kind? and for what?
1. Is a computer network and the Internet used in math lessons?		
Yes	25	
No	4	
2. What ICTs are available in school for supporting math teaching of students with visual disabilities:		
Equipment:		
laptops	17	
tablets	4	
smartphones	4	
interactive tables	16	
interactive monitors	4	
other (please specify)	1	*students' private laptops*
Software:		
for editing mathematical formulas	9	*LibreOffice, Inkscape*
for editing mathematical graphics	11	*LibreOffice, Inkscape*
For swell printing	9	
other applications (please specify)	7	*Duxbury for writing math text (Braille text translator)*
3. During math lessons students with visual disabilities use:		
laptops (what for?)	10	*For taking notes, writing content and solving problems. For writing down and archiving received information, work cards and lesson exercises. To search for relevant information on the web, for example, by using resources of e-textbooks for maths. Writing, reading texts, solving problems. They see the math tasks and problems in high enlargement and solve them. To solve problems in mathematics. 2x (for taking notes and reading them as well as for solving tests).*

tablets (what for?)	6	*As a calculator. They use multimedia programs, e.g. Klik uczy…(* Click teaches…)*. Manuals in the multimedia version. 2x (reading the contents of the tasks in the enlargement, viewing the lessons available on the Internet)*
smartphones (what for?)	3	*Calculations, finding information. To take pictures of the materials obtained during lessons or to partially record the teacher's remarks or lecture (only upon teacher's consent). Calculator function, taking pictures from the board. 2x (Performing calculations on the calculator, after taking a photo of the content of the problem written in the black-print - reading it enlarged). 2x (reading the contents of the exercises in enlargement, viewing the lessons available on the Internet)*
Students with visual disabilities do not use laptops/tablets/smartphones during lessons	13	
4. During the math lessons students with normal vision use:		
laptops (what for?)	8	*Notes, exercises from the teacher. For taking notes and archiving received information, work cards and lesson exercises. To search for relevant information on the web, for example, they use resources of e-textbooks for maths. To solve problems in mathematics. To draw graphs.*
tablets (what for?)	7	*Quizizz. Educational programs from the matzoo.pl website. 2x (Browsing through lessons available on the Internet, using the calculator).*
smartphones (what for?)	5	*Quizizz, calculator function. To take pictures of the materials obtained during lessons or to partially record the teacher's remarks or lecture (only upon teacher's consent). Searching for alternative solutions to problems and issues. 2x (Browsing through lessons available on the Internet, using as a calculator).*
Students with normal vision do not use laptops/tablets/smartphones during lessons	14	
5. During math lessons teachers use:		

laptops (what for?)	7	*Multimedia manuals, preparation of work, notes, tests. To give information to students in the form of a lesson presentation, to demonstrate various types of simulations of phenomena. To create learning aids. 2x (for interactive board), PREZI. To support the teaching process. Presentations, films. Especially in geometry - construction problems. For multimedia presentations. To display the content of e-textbooks. 2x (Connection to an interactive board and using it to screen examples).*
tablets (what for?)	4	*Preparing and conducting quizzes. For a presentation, they use educational programs. 2 x (Connection to an interactive board and using it to screen examples).*
smartphones (what for?)	5	*Calculations, finding information. Quick search for data from the Internet if the lesson requires data that were not foreseen. 2x (Creating exercises with QR codes)*
PC computers (what for?)	1	*Connecting to interactive table*
Teachers do not use laptops/tablets/smartphones during lessons	8	
6. Math teachers use ICT:		
to create student work cards with exercises (which ICT tools?)	26	*Office (Word, Excel), Internet. Various software. 3x (Office 365, GeoGebra), Paint, PlatMat Teacher, various software with superkid.pl. 3x Laptop. Tablet. Text editor. 2x LibreOffice. 2x (thatquiz.org, GeoGebra, animations from the Scholaris platform – Ambulans, Play with a Mirror).*
for creating mathematical graphics (which ICT tools?)	21	*Word, Corel, 4xGeoGebra. Various software. Paint. MSOffice. 2x Laptop. Graphics editor. 2 x LibreOffice Spreadsheet. 2 x (GeoGebra, Paint, graphics editor in MS Word, Excel, Gimp)*
when sending work cards and other mathematical content to the student (which ICT tool?)	11	*gmail, USB memory, email, Internet, 2x emails. 3xOffice 365. Laptop. 2 x (ONE DRIVE, cloud).*

to prepare mathematical tests (which ICT tool?)	18	*Ready-made tests, MS Office (Word, Excel), laptop, various software, Paint, PlatMat Teacher. klasowki.pl website. Printer (?). 2xLaptop. Text editor. LibreOffice. 2 x (Thatquiz.org, Moodle).*
to prepare arithmetic exercises using the written method (which ICT tool?)	14	*Ready-made exercises. SuperKid.pl spreadsheet generator. Laptop. Text and graphics editor, MS Excel. 2 x LibreOffice. 2 x (spreadsheets on superkid.pl website)*
in other situations (describe situations and used ICT)	3	*To write and print exercises used during lessons, tests in various forms, additional homework - Duxbury, mathematical drawings – simple ones are ready-made elements in Word, Corel. Creating a presentation. Teaching/Learning programming/coding. Mathematical dictations.*

Table 16 A summary of data from the survey on ICT applications in the mathematics education of students with visual disabilities

Source: own work

Questions and answer options	Number of answers	Collected answers to open questions: what? which? for what?
1. Are these tools used in maths lessons:		
Interactive board	15	*2x (e-Instruction), Esprit DT, 2x SMART, Qomo, Hitachi*
Interactive monitor	2	
Interactive boards and monitors are not used in maths lessons	11	
2. Are these tools used by students during maths lessons:		
laptops	10	*1 x (student owned)*
tablets	5	
smartphones	7	
laptops tablets and smartphones are not used by students during maths lessons	12	*Only PC (connected to interactive board)*

3. What is an interactive board or monitor used for in the maths education of students with visual disabilities?		
for displaying the content of exercises in enlargement	19	
for displaying content of exercises in a contrasting manner	14	
for solving tests:		
using touch gestures	1	
using QWERTY keyboard	1	
with touch gestures and QWERTY keyboard	3	
using Braille keyboard	0	
for group work while solving problems:		
low vision students	14	
blind students	1	
for multimedia playback	11	
for other use	2	*for creating, searching and collecting interactive educational resources. Interactive coding.*
4. Please indicate helpful functions using an interactive board (or monitor), assuming that teacher and students are equipped with computers:		
displaying on the board content created on a computer by the teacher	22	
displaying on the board content created on a computer by the student	19	
sending content written on an interactive board to the student's computer to be read by the student	17	
tests displayed:		
On interactive board	17	
sent to students' computers	13	
ranking of results displayed on the board	10	

the solution of a test by a student chosen by the teacher displayed on the board	15	

Table 17 A summary of data from research on the use of interactive boards and monitors in maths education of students with visual disabilities

Source: own work

4.3.2.1.4 Analysis of data from research on the use of ICT

Schools' network infrastructure and computerization

The majority of schools have network infrastructure and Internet that they use in mathematics lessons. The predominant computer equipment owned by schools and used for math classes are laptops (17/29) and interactive boards (16/29), interactive monitors which are more modern devices, but also more expensive, are less frequently used (4/29). Few respondents mentioned LibreOffice and Inkscape as the software available in schools. Both programs are used to edit formulas and mathematical graphics. Open source LibreOffice software consists of several office applications, among them the MATH formula editor, while the open source Inkscape software enables creation of vector graphics in the SVG format and can convert to many other formats. It is well adapted to creating mathematical graphics and placing symbols (e.g., the angle symbol), and also has Polish language ready-made aids to conduct lessons. The Duxbury Braille (https://www.duxburysystems.com/) translator software, which can be used with different printers and many Braille notations, and translates MS Word documents into Braille script, was also mentioned as school equipment.

Hardware and software used in math lessons by students with visual disabilities

The most frequent answer (13/29) indicated that no types of computer equipment such as laptops, tablets or smartphones were used in mathematics lessons. Over a third of students use laptops (10/29), tablets are less popular during the math lessons (6/29) and smartphones are not commonly used in math classes (3/29).

Students with visual disabilities use laptops mainly to make notes during lessons and write down exercises as well as classroom and/or homework assignment cards, solve problems, search for information on the Internet, and enlarge the mathematical content they are working on.

Tablets serve as calculator; they are also used to enlarge the content of the board; to browse multimedia textbooks and lessons on the Internet; to use visually attractive; multimedia educational programs; e.g. *Klik uczy…* (*Click teaches*) applies to younger students. This is a series of Polish programs for children up to 9 years old. In this series there is the program *Klik uczy liczenia* (*Click teaches counting).*

Smartphones, which are not often used in math lessons, serve as calculators; in searching for information and viewing lessons on the Internet; also to record lessons upon teacher's permission; to take pictures of supporting materials; and the content from the blackboard for reading in enlargement.

Hardware and software used in math lessons by students with normal vision

Similarly to students with visual disabilities, almost half of the respondents (14/29) claim that the students with normal vision do not use computer equipment in math classes, and they less often than the students with visual disabilities use laptops (7/29), which can be justified by the fact that a laptop is not necessary for sighted users to be able to record mathematical content. A laptop in the hands of a Polish student with a normal vision during a math class is not a common phenomenon among the students under study. Smartphones are even less popular (5/29).

Laptops are used by students with normal vision to take notes and store information received during lessons as well as exercises and assignment cards. Problem solving is also supported by laptops. With the help of laptops, students use e-textbooks and other Internet resources needed to solve mathematical problems. Students also use them to draw function graphs.

The use of computer equipment by teachers during math lessons

As it transpires from answers by teachers, they use computer equipment during math lessons to a lesser extent than their students with normal vision and those with impaired vision. This picture of the use of laptops/tablets/PC (total 12/29) is seemingly inconsistent with the results of research on the use of interactive boards and monitors, shown further on, which indicate greater use of tables and monitors - by more than half of respondents (17/29). It is worth noting, however, that interactive boards and monitors can work on their own, without connecting to computers, when using their own software, e.g. simple formula editors, or in the case of monitors – software previously installed or available in Windows 10 (pre-installed in interactive monitors).

Teachers use laptops to create and display to students - on screens or interactive boards - mathematical content, multimedia presentations and educational films as well as e-textbooks. Laptops are also used to create and display to students construction problems in geometry. Tablets are used for similar functions, but they have additional applications, in comparison to laptops. They have functions similar to smartphones, such as servicing exercises with QR codes, recently very popular in education, not just math teaching. The built-in camera, both in tablets and smartphones, is used to read the QR code(s), with a QR code reader, e.g. with a popular free of charge QR Droid reader. The read out QR codes are created by the generators as standalone graphic files, or pdf. QR codes are entered on the printout of the exercise or instruction. The information attained through QR code may be, for example, the correct solution or the next part of the exercise. An example of using QR-code is shown in Figure 14. This is an incomplete code, to be completed by the student by blackening the fields according to the numbers being the results of the calculations. After filling in the fields, the read out QR-code leads to the next part of the exercise.

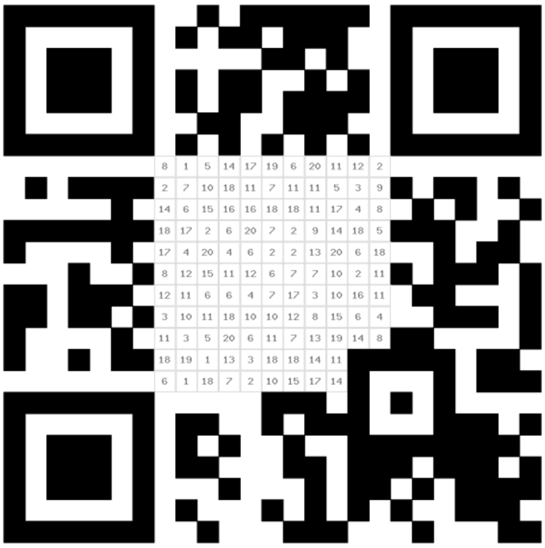


Figure 14 QR-code to be filled in with the results of calculations

Source: http://www.superbelfrzy.edu.pl

Thus, the smartphones serves the teachers in creating exercises/problems with QR-codes, mainly to test the correctness of the structure of the exercise, and they are used by students in solving these types of problems. Apart from this function, basically performed by teacher outside of the lesson, the smartphone is used by teachers to quickly search for information needed unexpectedly during the lesson and to calculate on the smartphone's calculator.

Creating student assignment card supported by ICT

The creation of assignment cards is clearly a computerized process (26/29). Most often, the teachers pointed to the MS Office package (Word and Excel) and the free of charge GeoGebra program (producer of the GeoGebra Team under the GNU GPL license). MS Word is used by teachers to create mathematical content and formulas, GeoGebra to create drawings of geometric figures and function graphs. In addition, teachers use Internet resources on such portals as scholaris.pl, the official MEN (Ministerstwo Edukacji Narodowej) portal with teaching aids, including the considered as useful Ambulans and Play with Mirror animations. The commercial superkid.pl portal is also used with teaching aids for all subjects, including mathematics, for younger students up to 10 years old.

One opinion referred to the PlatMat TEACHER application, which is worth emphasizing because this application is an innovation introduced by IMM as part of the PlatMat supporting technology, currently being developed (2018) by NASK PIB. In this application there are 3 formula editors, also an editor of simple multiple-choice tests.

Teachers have indicated the thatquiz.org/pl portal in Polish version, as a good tool for preparing mathematical tests. It also enables their printing and copying into a file, which is useful for creating assignment cards. The portal also offers the possibility of preparing and conducting on-line tests, which can be used during a lesson, for example to invigorate it. It is worth noting that there are several portals used by math teachers for creating and conducting mathematical tests, such as the commercial testportal.pl portal presented at the 3rd National Congress of Mathematics Teachers. After the test this portal provides statistics of results, though in our research the teachers of visually disabled students indicated the thatquiz.org portal.

The creation of mathematical graphics supported by ICT was declared by many teachers (21/29). The tools for creating mathematical graphics are first of all the GeoGebra. Teachers use also Paint (Windows application), Gimp (open source under the GPL license), a commercial CorelDraw package (produced by Corel Corp.). They also use simple graphical tools easily available in MS Windows and LibreOffice office package, offering basic graphical functions.

Providing electronic assignment cards or exercises, is done relatively rarely (11/29) in 4 following ways:

- by e-mail (indicated most often);
- through cloud (OneDrive);
- by USB;
- MS Office 365 package (and accompanying MS Exchange and MS SharePoint).

The creation of ICT-supported tests was declared by more than half of responders (18/29). Teachers use a wide variety of tools to prepare tests, most of them generally defined (ready-made, using various software, using laptop). The portals thatquiz.pl and klasowki.pl, the (LMS) Moodle e-learning platform as well as MS Office and LibreOffice office packages were mentioned by name. Paint and PlatMat TEACHER got one indication each.

In almost half of the answers (14/29), the respondents declared that they use ICT in teaching arithmetic operation in a written form. The most often used supporting tools are sheets created with the SuperKid sheet generator and sheets from the MS Office and LibreOffice office packages. The remaining answers did not identify the tools used.

ICT supporting tools used in other situations than those envisaged in the questionnaire (3/29), include the Duxbury translator of MS Word text into Braille script for convex printing of mathematical content (assignment cards, exercises, various tests) in Braille notation.

4.3.2.1.5 Analysis of data from research on the use of interactive boards and monitors

In more than half of the surveyed schools (17/29), teachers use interactive boards and monitors in maths classes. The vast majority of them (15/17) use interactive boards. Only in two cases, interactive monitors were indicated, which is consistent with the confirmed by research schools' IT equipment with decisively greater number of interactive boards. According to the respondents, schools where teachers do not use interactive devices of the board/monitor type during maths classes are in minority (11/29).

The availability of laptops, tablets and smartphones in maths classes depends on a school. In a very significant number of the examined schools (12/29), pupils do not use any IT equipment at all in maths classes. In the other institutions covered by the research, laptops are the most popular (10/29), smartphones are used less often (7/29) and then tablets (5/29).

According to the information provided by participants, the interactive boards and monitors in the mathematics education of students with visual disabilities are most frequently used for enlarging the displayed content of exercises for the low vision students – (19/29), with appropriate intensification of differences between colours, that is by increasing contrast - (14/29) and for group work, focused mainly on low vision students - (14/29). The participation of blind students in group work during lessons using interactive devices is negligible - only one teacher indicated this type of activity in his/her school. The boards are also often used for screening multimedia files (11/29).

The results of the research confirmed the limited use of interactive boards and monitors in the area of solving test problems. Only (5/29) teachers declared conducting tests using an interactive method, by touch gestures or QWERTY keyboard. It is worth noting that there was no school where tests were conducted in Braille using a Braille keyboard. In addition, the teachers confirmed the use of interactive boards and monitors to create, search for and collect multimedia educational resources, as well as to conduct interactive coding lessons.

The opinions of the respondents regarding the indication of functions helpful in teaching mathematics with the use of an interactive board (or monitor) focused mostly on displaying the content created on a computer by a teacher - (22/29), and content created on a computer by a student (19/29). Sending content stored on the interactive board to student's computer with the possibility of reading it by both the low vision and blind students, was also considered to be important (17/29). Teachers pointed out the need to support testing by using an interactive board, in particular for:

- screening the content on the board (17/29);
- sending to students’ computers (13/29);
- displaying on the board the ranking of results obtained by students (10/29);
- displaying on the board the test solution from a computer of a student selected by the teacher (15/29).

In teaching students affected by visual disability, by means of an interactive board, the automatic correction of their handwriting, displayed on the board, would also be useful, due to careless, distorted writing that is often illegible for both the teacher and other students in the classroom. The work of the teacher would be easier with the possibility of paging the content created on the board during the lesson, so that it could be scrolled, marked and text fragments relevant to the solution of the problem underlined. In addition, teachers indicated the need to display enlarged parts of the textbook for visually impaired students, as well as e-textbooks.

4.3.3 Conclusions from the research on the use of assistive ICT tools.

According to the results of previous surveys from 2014, teachers are open to computerization of the teaching process and it can be stated that they are fully computerized in the preparation of materials for mathematics lessons. This conclusion can be drawn indirectly on the basis of the declared ICT tools used to prepare material for lessons. The creation of student assignment cards using ICT tools was declared by almost all teachers (26/29), which, taking into account the low participation of teaching-computers in maths classes (12/29), is the result of the high level of computerization of teachers at home. Teachers prepare student assignment cards, exercises and tests using their preferred ICT tools. They use both the tools installed on their computers as well as from online resources to get ready-made materials, for example, animation making the lessons more interesting, or to prepare tests. Computerization of teachers as well as all students in the classroom for maths classes remains a problem.

The collected data shows that interactive devices of board/monitor type are used in the maths education of students with visual impairments, especially for low vision students, and are mainly used to display the content of exercises in an enlarged format and for group work of these students. Blind students are mainly recipients of sound replayed on a multimedia board.

The observed limited use of interactive devices for maths tests does not include students using Braille. The teachers did not confirm the use of Braille devices, e.g. Braille keyboard, for solving tests on the interactive board, but declared the use of gestures and the QWERTY keyboard.

Blind students are deprived of the possibility of precise handwriting of the content on an interactive board, while the graphics and formulas created by the teacher or low vision students on an interactive board are not available to them. As a result, they are deprived of fully active participation in a lesson supported by an interactive board. This uncomfortable situation of a blind student may be one of the reasons limiting the use of interactive equipment in maths education of students with visual disabilities. To a lesser extent, this applies to homogeneous classes attended only by low vision students, and it is more pertinent in case of mixed classes, attended by both low vision and blind students. The low level of computerization of students in maths classes is not conducive to solving this problem. Although the most frequently-used equipment in maths classes are laptops, those that are owned by schools as well as private ones, still the level of their use is far from adequate.

The interactive boards are provided with educational software, which is also dedicated to the needs of mathematics. It allows, among other functions, to create drawings and write mathematical formulas. Content on the board can then be exported to files in various formats, including HTML. Unfortunately, both drawings and expressions saved in html files are images (png, jpg) and not structural objects saved in MathML (formulas) or SVG (vector graphics), which means that they are inaccessible to a blind student. A note from a lesson created in this way cannot be very useful to a blind student.

All the participants have to be computerized or else the exchange of maths content and information between the teacher and students and greater availability of teacher's assistance in the classroom and outside the lesson are not possible.

Materials prepared by teachers students receive in printed form. Materials for low vision students are printed in in an enlarged format, while Braille printouts of assignment cards are obtained after converting into Braille script using Duxbury program and then printing. Typhlographic printouts with mathematical graphics related to problems are printed either on Braille printers (e.g. ViewPlus), or by using more and more popular technology – black ink printing of graphics on swell paper, and then heating it on the heater. The ViewPlus printer is about 10 times more expensive that a heater, hence the popularity of swell paper.

We can say that we have a computerized and digitalized process of preparing maths materials, but not process of teaching in the classroom. There is a lack of ICT used tools that, in particular, provide on-going support for blind student's work on mathematical problems, who have been so far been using Braille equipment during maths lessons (Braillers, Braille notebooks, which are not equipped with ICT tools supporting operation on mathematical content).

The low vision students benefit from four types of supports in the classroom:

- upon enlargement of screen content they are able to use the equations editor available in MS Word and make notes in this editor;
- they use printed materials in a larger font and monitors that magnify printed material or a student's notebook, also at the time when student writes in a notebook;
- they have, although not often, digital enlargers of content recorded with a camera from the board;
- they use cell phones to enlarge the content of the board or of textbooks, or to save it in a photo, and then display in an enlarged form.

However, this kind of support is insufficient due to the large diversity in the level of visual disability among visually impaired students, often requiring techniques and support tools designated for blind students, such as reading mathematical content with synthetic voice, the possibility to navigate through the structure of the formula with concurrent audio-reading of the formula elements.

There remains the problem of more intensive inclusion of ICT to support students in maths classes, which seems to require systemic solutions to meet the need for:

- computerization of maths classes;
- equipping students with laptops;
- training of both teachers and students in the area of ICT supports;
- introduction of ICT to mathematics education at the early stages of education, which the students themselves expressed in the research (discussed earlier in this paper).

4.3.4 Assessment and Examination

National information about junior high school and high school exams

Information on the examination and assessment of visually impaired students was collected on the basis of reports of the Central Examination Commission in Poland (CKE – Centralna Komisja Egzaminacyjna) in [48] [49] [50] [51] [52] [53] [54] on the results of junior secondary and secondary school exams from the last years, and on the basis of expert reports by mathematics teachers from the one sample SOSW for Blind and Low Vision Children.

Mathematics is a compulsory subject in the junior high school and final high-school examinations. The junior high school exam in mathematics assesses the level of junior high school students' mastery of the skills covered by the core curriculum in mathematics at the second and third stage of education. The final high-school certificate examination checks the extent to which the school-leaver meets the requirements concerning knowledge of mathematics covered by the curriculum of the general education for the fourth stage of education. However, the selected problems in the examination paper may, according to the principle of accumulation adopted in the core curriculum, refer to the requirements of the earlier stages (first, second and third).

The compulsory junior high school exam in mathematics students pass on one level - basic. The result of the junior final high school exam does not affect the completion of school and is not recorded on the school leaving certificate, but it is taken into account in the recruitment process for upper secondary schools. The final high-school examination in mathematics, as a compulsory subject, is taken at the basic level. If mathematics was chosen by a student as an additional subject, the exam is also taken at the extended level.

The core curriculum in mathematics at all levels of education is the same for students with no visual disability and visually impaired students. Therefore, all mathematics examination papers for visually impaired students contain the same set of problems as the standard papers, but in a form adapted to the type of disability of the student who has a special educational need certificate issued due to disability.

The low vision students receive examination papers with adapted font size (Arial size 16 and Arial size 24 respectively) and simplified and enlarged graphic forms. For blind students, the paper is prepared in Braille with convex drawings. Since 2016, drawings for the blind, in addition to being included in the content of the paper, are provided in the form of a separate booklet.

The examination procedure also includes adjustment of the examination conditions so as to minimize the limitations resulting from the student's disability. It consists of, among other facilitating implements: using the appropriate specialist equipment (Braille writing machine, specially adapted computer, cubarithm, foil with drawing accessories, optical devices); providing additional lighting to the low vision student, extending the time of examination by 30 minutes. It also ensures, during the exam, the presence and help of a teacher assisting the student in reading commands and texts and/or in writing the examined student' answers on sheets attached to a black-print paper. If the examined student uses the assistance of a supporting teacher, the exam must be recorded using a sound recording device. The sound recording is an integral part of the examination paper. This way of passing the exam is intended for students who, for example, lost their sight entirely or for those situations where it became significantly worse during secondary school education and they did not master braille techniques at a level allowing them to work independently on the exam paper.

The majority of blind and visually impaired students do not use computers in maths exams, although there are isolated cases when this type of support is applied.

In 2017, the standard maths exam paper at the junior final high school exam contained 23 problems, including 20 closed problems and 3 open ones. Among the closed problems, the majority were problems with multiple-choice answers, out of which one had to choose one answer, in five true-false questions - assess the truth of sentences, and in one - answer the question and indicate the correct justification. Open problems required junior high school students to formulate their own solutions. The problems had a table, drawings and graphs.

The final high school exam consists of three groups of problems:

- Group I contains closed problems. Four answers are given for each problem, of which only one is correct. The student indicates the correct answer on the answer card. Each answer in this group is scored on a 0-1 scale.
- Group II contains open problems requiring a short answer. The student gives a short justification for his/her answer. Problems from this group are scored on a 0-2 scale.
- Group III contains open problems requiring an extended answer. These problems demand careful planning of the solution strategy and presentation of the reasoning method. They are scored on a 0-4, 0-5 or 0-6 scale.

In 2017, an exemplary final high school exam paper in mathematics at the basic level consisted of 25 closed problems with multiple-choice answers and 9 open problems, including 6 with short answers and 3 demanding an extended-answer. The problems checked the knowledge and skills described in the five areas of the general mathematics core curriculum: use and creation of information (five closed problems); use and interpretation of the representation (fourteen closed problems and one open problem demanding short answer); mathematical modelling (five closed problems, three open problems with short answers); creation and use of strategy (one closed problem, three open problems with extended answers); and reasoning and argumentation (two open problems with short answers).

Exam papers in mathematics are increasingly universalized and only to a small extent require adaptation of problems to the needs of candidates with visual disabilities. In 2015-2017, in final maths papers at the basic level, covering 34 problems each, adaptive changes were made only in 9 problems. They consisted of, among other adjustments:

- deletion of drawing and replacing it with a verbal description;
- simplification of drawings
    - transferring the values of angles written in the drawing to the content of the problem while replacing them with symbols in the drawing,
    - replacing the drawing of a projection of cylinder by its cross-section;
- completing the problem with additional data – to the graph of a quadratic function there was added a table of its values for selected independent variables;
- replacing simple fractions in the content of the problem with fractions in decimal form.

The examination papers of blind and low vision students are assessed according to the same criteria as those by students without disabilities.

Students	2017	2016	2015
low vision and blind	795	764	733

Table 18 Students solving problems in adapted papers at the junior high school maths exam

Source: Own work based on [48] [49] [50]

The data of the Central Statistical Office (CSO) collected in Table 19 shows that the number of students with visual disabilities in upper secondary schools in Poland in the school year 2015/2016 increased by 12% compared to the school year 2013/2014. The data in Table 20 indicate a similar increase, by 13%, in the number of disabled students in the final high school grades, which may mean that students continue their education in upper secondary schools until the final exams. CSO reports do not provide detailed statistical data on the high school students sitting for the final high school exam, in relation to the type of disability, including visual impairment, but refer to all the disabled students (Table 20). Based on the Central Examination Commission (CEC) data contained in Table 21, it is known that in the same period of three school years under study, the number of visually impaired students who sat for final high school exams increased by 11%, however, due to the lack of detailed data, it is not possible to determine what the percentage of students was who took the final high school exam in the group of secondary school graduates with visual disabilities (*in Poland you can be a upper secondary school graduate without passing the final high school exam*). Information on this subject, using as an example data from one Special School and Educational Centre, is presented in the further part of the paper.

	School year 2015/2016		School year 2014/2015		School year 2013/2014	
School type	**blind**	**low vision**	**blind**	**low vision**	**blind**	**low vision**
High schools	17	495	11	424	16	415
Special high schools	31	124	34	128	40	151
Technical schools	5	408	4	322	3	269
Special technical schools	35	153	37	157	45	196
Sum total in all secondary schools	**88**	**1180**	**86**	**1031**	**104**	**1 031**

Table 19 Blind and low vision students in public and special secondary schools in Poland

Source: own work based on CSO Education and Upbringing reports [55] [56] [57]

	School year 2015/2016	School year 2014/2015	School year 2013/2014
School type	Disabled graduates		
High schools	1 272	1 175	1 037
Special high schools	321	351	273
Technical schools	248	232	253
Special technical schools	75	50	128
Sum total in all secondary schools	**1 916**	**1 808**	**1 691**
Sum total in all high schools	1593	1526	1310
Sum total in all technical schools	323	282	381

Table 20 Disabled graduates in public and special secondary schools in Poland

Source: own work based on CSO Education and Upbringing reports [55] [56] [57]

Maths exam	**Students**	**School year 2015/2016**	**School year 2014/2015**	**School year 2013/2014**
basic level	low vision	348	303	310
	blind	25	19	25
	others	738	622	667
extended level	low vision	63	41	33
	blind	3	3	3
	others	173	124	81
Sum total on basic level	all disabilities	**1111**	**944**	**1002**
Sum total on extended level	all disabilities	**239**	**168**	**117**

Table 21 Students solving problems in adapted papers at the final high school exam in mathematics

Source: Own work based on [51] [52] [53]

	School year 2015/2016	School year 2014/2015	School year 2013/2014
School type	Disabled graduates		
High schools	652	629	515

Special high schools	94	93	102
Technical schools	182	139	118
Special technical schools	38	31	29
Sum total in all secondary schools	**966**	**892**	**764**

Table 22 Disabled graduates of public and special schools in Poland who have obtained a secondary school-leaving certificate

Source: Own work based on [55] [56] [57]

From the CEC data contained in Table 21, it transpires that the group of blind and low vision students sitting for the final high school exam in the period under study constitutes about 33% of all students who solve problems on the adapted sheets at the obligatory mathematics exam. Not every type of disability requires the adapted high school exam sheet. A certain percentage of students with disabilities use the standard examination sheet, which is why the data in Table 21 may not include all students with disabilities sitting for the final high school exam in a given school year. The comparison of the Central Statistical Office's information on disabled graduates of upper secondary schools with CEC data on the number of disabled students sitting for final high school exam using adapted examination sheets allows us to state that in 2014-2016 more than half (56%) of all disabled graduates of upper secondary schools sat for the final high school examination, and 85% of students in this group passed it successfully and received a secondary school-leaving certificate. Due to the possible underestimation in the CKE reports of the number of disabled students who sat for final high school exams, as mentioned above, the ratio of students with disabilities sitting for the final high school exam may be higher, and the actual exam pass rate may be lower.

Only a few among those with visual disabilities who sit for final high school choose the final exam in mathematics in the extended formula, which is shown in Table 21. The percentage of students sitting for the exam in the extended formula in both groups is similar - blind - on average 13% in 2014-2016, and low vision students - on average 14% in the same period. The lack of interest in this level of the exam results from the fact that student has to achieve higher level of mathematical competence, while visual disability is a very serious impediment.

Detailed information about high school exams on the example of the chosen SOSW

The referred to CEC (Central Examination Commission) reports contain no information about the rate of passing junior high school and final high school exams by students with visual disabilities, which is of interest of the authors of the study. These data are not easily available. It is also difficult to obtain data on the use of supporting ICTs by the blind and visually impaired students on final high school exams. To provide the minimum information to the authors and readers of a study in this area of knowledge, at least based on sample data, in May 2018, NASK PIB commissioned experts from one of the largest special centres in terms of the number of schools[3] and students, to prepare two studies. One study concerned the analysis of the accessibility of final high school exam problems from 2015-2017 and the availability of ICT supports for the student at the final high school exam[4]. The second study constituted an extension of the conclusions and recommendations of the CEC from the final high school exams in the years 2015-2017[5], from the perspective of the SOSWs (Special School and Education Centre) experience.

Maths exam	**Students**	**2017**	**2016**	**2015**
basic level	low vision	21	10	7
	blind	3	5	6
extended level	low vision	2	0	1
	blind	0	0	0

Table 23 Students of technical school solving tasks in adapted papers in the final exam in mathematics at the sample SOSW for Blind and Low Vision Children

Source: Own work

In total 62% of blind and low vision students at SOSW sat for final high exam in 2015-2017, with a slightly higher percentage in the group of blind students (64%) in relation to the low vision students (60%).

In 2015, the final exam in mathematics at SOSW was passed by 54% of students and was 16 percentage points lower than the pass rate of students with vision impairments in secondary technical schools in Poland. In 2016, it amounted to 47% and was lower by 28 percentage points than the overall pass rate in Poland. In the following year, 2017, 72% pass rate was achieved at SOWS, only slightly lower than the average math exam pass rate in technical schools in Poland, which amounted to 79%.

In 2015, the average result of the final high school exam in mathematics at SOSW amounted to 44%, whereas the overall average result in Poland in this type of school within students with vision impairments was 1 percentage point lower, that is 43%. The best students got high marks, at the level of 80%, 92% and 94%. In the following year, the average mark at SOSW was 34%, while in Poland the average mark obtained by technical school leavers was 12 percentage points higher, that is 46%. In 2017, the average result of the final exam in mathematics in SOSW was 40%, so it was 6 percentage points higher than the average result at SOSW in 2016, and 5 percentage points lower than the average result for all technical schools in Poland, which was 45%.
According to teachers, both the number of students sitting for the final high school exam and the passing rate of this exam in different years vary and depend to a large extent on students'

[3] The centre includes: Primary School, Junior High Schools, Technical School, School of Industry, Work Training School, Post-Primary School, High School for Adults, First Level School of Music

[4] *Analysis of problems in the accessibility of mathematical problems from high school final exams at the basic and extended level from the last 3 years, adapted in terms of form [without changing the content] and replacement tasks [with changed content] for blind and low vision students, and assessment of the possibility of providing them with ICT supports*

[5] *Additions to the conclusions and recommendations contained in the CEC reports from the final high school exam in mathematics from the last 3 years concerning the results of a) blind students, b) low vision students based on the high school final exams at the basic and the extended level in the Special Upbringing and Education Centre*

intellectual abilities, on the amount of work they put into learning and on their involvement, as well as on general health, affecting systematic participation in school activities.

In the last three years at SOSW, only two visually impaired students used computers in the high school maths exam. In the first case, the student on his own read the content of the paper placed under the enlarger connected to the computer, then solved the problems manually and wrote the results with a pen on the answer card. In the second case, the student had an adapted electronic version of the examination paper in pdf format and used the assistance of a supporting teacher in reading and writing. While the content of exam paper was read by the teacher, the student at the same time watched it on the computer screen. It is worth noting that in both of these cases the computer did not serve the student to write solutions in digital form, but only assisted in reading the paper.

Having analysed the results of the final high school exam in mathematics at the basic level in the SOSW in 2015-2017, we can indicate the areas of skills and knowledge, which the blind and low vision students learn with greater ease than others. These include problems in the area of, among others:

- elementary concepts of statistics;
- properties of the arithmetic progression;
- properties of geometric figures;
- reading simple function graphs.

Most of the high school graduates were able to correctly solve problems requiring the use of a specific formula and particular skills covered by the core curriculum.

The problems with which the students had difficulties were from different parts of the curriculum, so there is no area that can be clearly identified as particularly neglected and requiring increased intervention. Difficulties were noted in problems related to, among others:

- mathematical modelling and executing percentage calculations;
- using and interpreting the representation;
- knowledge of elements of descriptive statistics, theory of probability and combinatorics;
- using and creating strategies with simple relationships between trigonometric functions;
- showing dependencies between the fields of figures presented in the drawing (blind students);
- solving equations or set of linear equations with two unknowns (blind and low vision students);
- trigonometry;
- planimetry;
- solid geometry;
- sequences and abbreviated multiplication formulas;
- roots and principles of operations on roots;
- the use of logarithm's definition and formula for the logarithm of the product, the logarithm of the quotient and the logarithm of the power with the natural exponent.

A common problem in the maths exam for blind students is the correct reading and interpretation of a drawing, without which it is not possible to solve the problem. Similar issues are caused by problems where a well-drawn graphics plays a key role, as it allows seeing certain dependencies. Blind students can then rely only on the imagination. Very often, students do not attempt to solve tasks with a ‘show’ or ‘justify’ command requiring a proving process in algebra or geometry. The errors in calculations made by examinees at various stages of solving the problem, which often leads to the contradictory results is another important obstacle.

The results of the final high school examination indicate that the problems solved with good results include, above all, those that do not require too many stages in the solving process or careful choice of strategy. The most difficult are the problems that require the development and implementation of a strategy composed of several stages.

4.3.5 Access to Formulas for Blind Students

Polish school textbooks use Braille mathematical physical and chemical notation (BNM), described in detail in the collective work edited by Jan Świerczek [58]. It is based on the International Mathematical Notation of Helmut Epheser, the so-called Marburg notation [59]. It enables writing expressions covered by the core curriculum of math and science subjects to the level of higher education, using six-dot braille.

The six-dot system limits the number of characters to 64, which makes it necessary to precisely and consistently apply specific rules in mathematical notation. The same sign can have many meanings, depending on the context. An important role in the Braille writing is the sequence of characters. Particularly important information is also expressed by the lack of a character, the so-called ‘empty character’ (separation, space). Most of the black-print mathematical symbols are represented in Braille using two or three characters. For writing more complex expressions and formulas in Braille, the level indicators (upper and lower) as well as special brackets and key characters are used. The basic level, otherwise known as zero level, has symbols on the text line. The consecutive levels are defined both up and down recursively.

Reading Equations for blind Students

Blind students in Polish schools learn mathematics in Polish Braille notation BNM (Brajlowska Notacja Matematyczna) using such Braille devices as Braille machines (Braillers) and Braille printers. Materials for mathematics lessons they receive in a tactile form. These are Braille and typhlographic prints as well as tactile educational aids such as cubarithm for learning arithmetic operations, folded models of solids and their projections. As a rule the formulas are read from Braille printouts. Students who have private note takers, take notes from lessons in Braille, entering the formula in BNM.

Writing Mathematical Formulas for Blind Students in Poland

Blind students write formulas and modify them on Braille machines. The dictated content they write on Braille machines. Correction of errors on Braille machines, according to the opinion of teachers from one of the surveyed SOSW, it does not take more of the student's time than in the digitisation of the edition. With six-dot notation they cross out incorrect entries, and write the correct answer underneath. They can take notes using private computer equipment. This happens among older students.

4.3.6 Access to Mathematics for Visually Impaired Students

Visually impaired students have: magnifying loupes in the classroom; monitors enlarging objects lying on the table; monitors magnifying the content of the board; interactive boards for displaying content in enlargement and by contrast. The low vision students write in notebooks, although there are problems with reading the script, because they have trouble keeping the horizontal and the vertical alignment of characters, which is very important when conducting arithmetic operations and solving equations. Some older low vision students use laptops (private or school-owned) when these are available to them, in particular the equation editor in MS Word.

4.3.7 Access to Diagrams for Blind and Vision-Impaired Students

Mathematical graphics are available to blind students in the form of typhlographic prints, on Braille or swell paper, prepared by the teacher or purchased. The Polish Association of the Blind sells tylphographic drawings and provides typholgaphic printing services. The drawings may present, for example, mathematical functions and diagrams, statistical charts, and various cross-sections. Students get to know figures, solids and function graphs also with the help of special tactile educational aids, such as solid models, 3D printouts of function graphs, plates for convex drawing with a stylus. The variety of educational aids in learning mathematical graphics also depends on the inventiveness and ingenuity of the teacher. An example of this is the use of a cork board, string and nails (pins) for obtaining various shapes of geometric figures.

These techniques are also available to students who do not know Braille.

The low vision students learn mathematical graphics in enlarged print or image. They draw it in notebooks using magnifying devices.

4.4 Assistive ICT for inclusive mathematics education in neighboring countries

The information contained in this chapter on the state and problems of computerization of inclusive education in Slovakia, the Czech Republic and Germany were collected on the basis of available publications.

4.4.1 Czech Republic and Slovak Republic

The national report [60] states that there is no national strategy in the Czech Republic regarding the use of ICT to support inclusion in mainstream classes. Assistive technologies that are widely recognised have not been broadly adopted in the Czech Republic yet. Unfortunately the majority of special needs teachers has not recognized the usefulness of mobile devices for SEN (Special Educational Needs) education yet. A number of Czech universities run special centres that support university students with special needs. One of the biggest is the Teiresias Centre (the official name is the Support Centre for Students with Special Needs) of Masaryk University in Brno. Since 2001, the Centre has been entrusted with the printing of the tactile version of the state secondary school leaving exam for the blind, and the centre also provides the National Comparative Exams for Students with Visual and Hearing Disabilities, especially for those preparing for the entrance exams to universities. Czech mathematic notation is based on Nemeth code. In 1995, a preliminary concept of a standard was accepted. A handbook for transcription from printed text to Braille was written based on the standard. Part of the information we have gathered about the state and problems of computerization in inclusive mathematics education in the Czech Republic and Slovakia was included in the article by Wiazowski [61]. The information was obtained mainly from Regec [62], Mendelova, Lecky [63]. Vojtech Regec of Czech Republic investigated potential barriers in access to digital information among students with visual impairments. Their findings showed that over the years the situation has significantly improved in various educational areas putting BLV students on a par with their sighted peers. It appears though that math and other areas of STEM still require a lot of attention [63]. He particularly focused on their access to math education and the level of awareness among special education providers of what types of digital options are available (2014). Despite the availability of special tools and formats as well as highly skilled support at Masaryk University, only about 30% of trained teachers of the blind and visually impaired in Czech Republic know of high end accessible math solutions. He quotes one of their survey respondents who stated that it is not just lack of awareness but in fact an unwillingness to reach out for digital tools. The teachers would rather attempt some forms of tasks modifications to eliminate those most challenging ones. It means that BLV students are deprived of opportunities to study more advanced math.

According to Regec [62] teachers in Czech schools have a wide range of tools at their disposal. The Blind Moose editor developed at the Masaryk University in collaboration with the Teiresias Centre is a MS Word set of macros for entering and editing math expressions in a linear format. It is accessible for all students regardless of their vision condition. The drawback of this solution is a lack of internal TeX and MathML converters. Despite certain shortcomings, The Blind Moose is praised by the author due to the native implementation of the Czech Braille code. LAMBDA is yet another conversion application teachers have at their disposal. This is a tool developed as part of an international project. The acronym stands for Linear Access to Mathematic for Braille Device and Audio-synthesis. The LAMBDA code can be presented in 8-dot Braille on the Braille display and visually translated into a standard print notation. Occasionally MathType and Math Equation Editor within MS Word are used to compose math worksheets.

Most of the Slovak blind and visually impaired children attend special primary schools. In mathematics lessons they are using Braille books with tactile pictures, to make notes they use electronic notebooks and for calculations mechanical typewriter. The disadvantage of typewriter is firstly that the way to get result of calculus takes too long, so pupils try to calculate in their minds and secondly notation of the calculations is too verbiage, so after a while the pupil is lost. There are special high schools for visually impaired students in Slovakia, but mostly oriented towards music, some handicrafts, etc. If a student wants to come into contact with mathematics then s/he needs to attend "normal" high school.

Because teachers of these schools are not specially educated in this field they often have to use the ”trial and error” method to find out the best way of teaching their blind students who are only integrated among sighted students. Blind and visually impaired students also encounter a lack of textbooks and study material and limited Braille notation for maths.

4.4.2 Germany

In Germany there are separate schools (Förderschulen or Sonderschulen) for students with moderate to severe learning disabilities, blind or deaf students, or those with physical disabilities. This practice, which puts some 430,000 German students in special, separate schools, has been criticized for not meeting the 2008, EU-ratified UN Convention on the Rights of Persons with Disabilities [19], which calls for a more inclusive, integrated education for disabled students. Critics say that by separating special-needs students from the general population, the German special education system fails, in that it puts disabled students at a disadvantage and prevents their integration into daily life. This is especially true for students with physical disabilities. Only in a few places in Germany are there some special-needs students integrated into regular schools.

In Germany there are problems resulting from the use of diverse Braille notations. Depending on the region of Germany, the following notations are used: Marburg notation; Stuttgart Maths Notation; and ASCII Maths Notation (AMS) called Karlsruhe notation. Both in schools and at universities, linear notation of formulas available in the LaTeX language is used. LaTeX saves formulas and other text in ASCII's 7-bit characters, Figure 15. This means that both formulas of ASCII and LaTeX are black and white and can be clearly represented in Braille (in a single document).

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

```
x_{1,2}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}
```

Figure 15 An example of a formula in LaTeX notation

Source: Own work

There are many free editors for creating texts in LaTeX, also texts containing formulas. There are also converters from LaTeX for Braille notation. The Jaws screen reader reads semantically in German the formulas stored in MathML, and the scripts convert the entry from LaTeX into MathML.

4.5 Discussion

In Poland, in education of blind students, the Braille system in the norm. It consists of a staff of teachers familiar with Braille and Braille notation, Braille devices, Braille textbooks and Braille-marked educational aids. The Polish educational system lacks systemic solutions imposing:

- the necessity to recognize the needs of both teachers and students in the area of constantly developing new ICT supports;
- the requirements to use of selected ICTs to assist the blind and low vision student in his/her mathematics learning process.

The problem concerns the support using ICT tools of a blind student who knows Braille, who is learning in special schools as well as, perhaps most importantly, blind students who found themselves in a mainstream school. A common practice in such schools dealing with blind students in maths classes is that they are left alone or, in other words, they are marginalized, because math teachers, without ICT support and not knowing Braille, are incapable to help them. The blind student in mainstream schools has to rely primarily on the oral transmission of mathematical content.

Low vision students who, with disability certificates, find themselves in integrated classes, are in a better situation in terms of the facilities at the disposal of public schools as well as teachers' preparation. If the degree of visual disability is not drastic, the students manage to learn mathematics using the equipment in the integrated classes. Otherwise, the ICT supports become necessary, for example, the techniques used to support blind students, including synthetic speech. The first attempts to support low vision students with a high degree of visual disability with tools developed for blind students were carried out in the 2016/2017 school year, during the pilot project implementation of the PlatMat tools in the three educational institutions mentioned earlier. This resulted in positive opinions, which were confirmed by written recommendations.

However, there were some weak, unfinished functions in PlatMat. Among them was a graphics editor and its limitations in the creation of drawings and geometric tasks. Students complained about the unintuitive operation of some graphics editor functions, misleading and extended drawing time. They postulated improvement of the user interface and functional improvements to the graphics editor. Teachers have expressed the opinion that blind pupils are not advised to read graphics on the touch screen, because the students use both hands to read convex graphics, while graphic navigation on the computer touch screen is done with one finger. This is the difference between student's works in the classroom and what he is used to. In their opinion, this tool does not fully replace the previously used way of reading graphics by blind students, but it can be a tool supporting the recognition of mathematical drawings and diagrams through the use of sound, which also engages the sense of hearing in the cognitive process. According to the students, the mechanism of navigation on geometric drawings requires improvements in the area of audio information that helps the student to orientate and locate particular elements of the drawing.

In summary, in Poland, as a result of research projects, the offer of ICT tools supporting the teacher as well as blind and low vision students in mathematics education has been created. It culminates as a suite of PlatMat tools available on the www.platmat.pl portal.

In order to enable the implementation of ICT support tools in a wide educational practice, both in mainstream education as well as in special schools the teachers should be first of all encouraged by systemic solutions. These solutions would relate teachers' cognition of IT tools, confirmed by a certificate, to their professional advancement. It would be needed a concurrent evaluation of educational institutions in terms of increased computerization of the educational process and the resulting effects.

5 Conclusions

In this report, an overview of the educational situation pertaining to mathematics, in the context of blind and visually impaired people, has been described. Now that all of the circumstances in each of the respective countries has been outlined, it is necessary to discuss what the common issues are, and how this affects the development activities proposed for the EuroMath project.

Though the problems manifest themselves in different ways, there are various common themes which transcend national borders. The investigations carried out by the three partners have garnered information which highlights the need to drastically increase the numbers of blind and visually impaired people studying mathematics. In order to do this, both subject-teachers, and those with responsibility for supporting the learner in the learning, and use of, their Assistive Technology, need to be made aware that it is feasible for a blind or visually impaired student to study mathematics. This report has shown that there are, on rare occasions, students who have managed to complete the courses of study and attain their goals. These achievements should be used as exemplars to motivate students and teachers alike.

From the technical standpoint, the research shows several things which must be catered for by any tools developed in the EuroMath project. These are:

- The ability for students to easily navigate equations, without an undue cognitive load;
- The ability for students to manipulate, and thus solve, mathematical equations;
- The ability for the materials to be rendered both digitally, and in the case of Braille, in hard-copy;
- The ability for teachers to quickly prepare worksheets and other learning objects, and efficiently disseminate them to their students
- The ability to exchange data with other applications currently in use in the various countries.

It is beyond the scope of this document to delve deeply into the technical requirements of the software, however the points given above can form a succinct set of guiding principles. What is equally important, is the production of the three-hundred best-practice examples which can be made available to teachers. This will, it is hoped, enhance the ability of these individuals to impart the mathematical knowledge to their students. As stated in the opening chapter, both the technical, and pedagogical challenges must be overcome if EuroMath is to be a success, and give blind and visually impaired people the educational opportunities possessed by their sighted peers.

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